TOWARDS A MORE RELIABLE APPLICATION OF HOT-WIRE ANEMOMETRY IN COMPLEX COMPRESSIBLE FLOWS

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ABSTRACT

In high speed ($0.3 < M < 1.4$) and low density ($Kn_w > 0.01$) flows, a hot wire anemometer is highly sensitive to the velocity, density and total temperature of the flow, making its application to these flow regimes complicated. In the present investigation, the HWA sensitivities to the aforementioned flow properties are computed using a data-reduction technique based on empirical correlations. A set of data obtained at different flow conditions is employed to demonstrate the concept and to discuss the application of the HWA technique to different flow ranges. To evaluate the methodology, a comparison with experimentally defined sensitivities available in the literature is performed.

NOMENCLATURE

d_w	Wire diameter
E_b	Bridge output voltage
G_r	Grashof number
I	Current
k	Thermal conductivity
l	Active wire length
Kn	Knudsen number
M	Mach number
Nu	Nusselt number
Pr	Prandtl number
Re	Reynolds number
R_l	Lead's resistance
R_t	Bridge top resistance
R_w	Wire resistance
S_{T0}	Sensitivity to total temperature
S_u	Sensitivity to velocity
S_a	Sensitivity to yaw angle
S_β	Sensitivity to pitch angle
S_ρ	Sensitivity to density
$S_{\rho u}$	Sensitivity to mass flux
T_0	Total temperature
T_r	Recovery temperature
T_w	Wire temperature
u	Velocity

Greek

a	Yaw angle
β	Pitch angle
η	Recovery factor
θ	Overheat ratio $\theta = T_w/T_0$
μ	Dynamic viscosity
ρ	Density
τ_w	Overheating parameter $\tau_w = (T_w - \eta T_0)/\eta T_0$
φ	Correction factor

Sub- and Superscripts

0	Total conditions
$1,2,3$	Wire number
$corr$	Corrected for compressibility effect
r	Recovery
w	Wire
$()'$	Fluctuating quantity
$\overline{()}$	Time averaged quantity

Abbreviations

CCA	Constant Current Anemometry
CTA	Constant Temperature Anemometry
HWA	Hot Wire Anemometry

INTRODUCTION

The Hot-Wire Anemometry is among the most popular techniques for measuring velocity fluctuations, and therefore turbulence, thanks to its high spatial resolution (wire diameters as small as 5 μm), and high frequency response (up to 50 kHz) [1]. It has been extensively used for many years, with the first works dating back to the beginning of the 20th century (e.g. Boussinesq [2] and King [3]). Despite being a widely used tool for turbulence research in incompressible and supersonic flows, few studies exist in high subsonic/transonic and low density flows. Its application is complicated in these flow regimes and still limited, even today. Turbomachinery flows often lie in this "problematic" flow range and researchers like Cukurel et al. [4] and Boyle et al. [5] have proposed extensions to the applicability of HWA in such conditions. Nevertheless, further investigations are required before a general measurement methodology is established.

In this context, the present investigation seeks to contribute to the understanding of the behavior of CTA hot-wire anemometers in different flow conditions, through the evaluation of their sensitivities to the velocity, density and total temperature of the flow. The sensitivities are computed using a data reduction procedure based on empirical correlations.

HWA OPERATING PRINCIPLE

The HWA technique is based on the convective heat transfer over heated wires. A HW is sensitive to the convective power of the flow that is investing it. In incompressible, isothermal flows the output voltage of the anemometer can be directly related to the flow velocity and well known relationships like King's Law can be used. However, in the most general case the convective heat transfer is expressed by the Nusselt number and depends on the following parameters [1, 6]:

$$Nu = f(Re, M, Pr, Gr, l/d_w, \tau_w, a, \beta) \qquad (1)$$

For Constant Temperature Anemometry (CTA), with a given probe geometry ($l/d_w = ct$), in a specific fluid with moderate flow temperature variations ($Pr = ct$), and by neglecting the natural convection (when $Re > Gr^{1/3}$) [1], the relationship reduces to the following equation [4]:

$$Nu=f(Re, M, \alpha, \beta) \qquad (2)$$

The particularities and limitations of the HWA application in different flow conditions are well summarized in a review paper by Stainback & Nagabushana [6], where the following flow regimes are distinguished (in terms of Mach number):

- subsonic incompressible flow ($M < 0.3$)
- subsonic compressible, transonic and low supersonic flow ($0.3 < M < 1.4$)
- high supersonic flow ($1{,}4 < M$)

within which, one can distinguish the following sub-regimes (in terms of Knudsen number):

- continuum flow ($Kn < 0.01$)
- slip flow ($0.01 < Kn < 0.1$)
- free molecular flow ($0{,}1 < Kn$)

The Knudsen number is defined as the ratio of the molecular mean free path length to a characteristic length of the flow (the wire diameter in the Hot Wire case) and it expresses the deviation from the continuum flow. It can be related to Mach and Reynolds numbers by:

$$Kn = \frac{M}{Re}\sqrt{\frac{\gamma\pi}{2}} \qquad (3)$$

In incompressible continuum flows the Nusselt number is a unique function of Reynolds. The same has been observed for supersonic continuum flows with $M > 1.4$ [6, 7], so in this case eq. (2) reduces to $Nu = f(Re, \tau_w)$. In high speed flows there is a strong Mach number effect, as expressed in eq. (2). Moreover, it has been observed by different researchers, that in low density flows this compressibility effect can extend much lower than the typical incompressibility limit of $M = 0.3$ [6]. Spangenberg [8] was the first to attribute this effect to gas rarefaction: at low densities the HW often operates in slip flow conditions due to its small dimeter. Thus, high subsonic/transonic continuum flows and subsonic slip flows can be treated together, as the same heat transfer relationship applies to both: $Nu = f(Re, M, \tau_w)$. Since Re, M and Kn are linked, as shown in eq. (2), any two of the three parameters can be used to describe the non-dimensional heat transfer relationship. So eq. (2) is equivalent to $Nu = f(M, Kn, \tau_w)$ and $Nu = f(Re, Kn, \tau_w)$. Different sets of variables have been used by different researchers, as described in Ref. [6].

The recovery factor η which defines the ratio of the recovery temperature to the total temperature of the flow is also a function of the flow conditions [1]. While in incompressible continuum flows its effect is negligible ($T_r \approx T_0$), it should be instead taken into account in compressible and slip flows. An empirical correlation with a wide range of validity in the form of $\eta = f(M, Kn)$ was proposed by Dewey [9]. It is based on a collection of experimental data by different researchers and can be used for all Mach numbers, while extending from the free molecule limit to the high Reynolds continuum limit.

Taking into account the recovery factor effect, the heat transfer balance over the wire is given by:

$$I^2 R_w = \pi l k (T_w - \eta T_0) Nu \qquad (4)$$

By expressing the power supplied in terms of bridge voltage, the Nusselt number can be rearranged as:

$$Nu = \frac{E_b^2}{k(T_w - \eta T_0)} \frac{R_w}{\pi l (R_t + R_l + R_w)^2} \qquad (5)$$

Kovazsnay [7] and Morkovin [10] put the basis of the application of HWA for fluctuation measurements in supersonic and transonic flows respectively. By assuming small perturbations, Morkovin [10] related the fluctuations of the output voltage of the anemometer to the fluctuations of density, velocity and total temperature of the flow for Constant Current Anemometry (CCA). For Constant Temperature Anemometry (CTA) and straight wires mounted parallel to the flow, the following relationship stands:

$$\frac{e'}{\bar{E}} = S_\rho \frac{\rho'}{\bar{\rho}} + S_u \frac{u'}{\bar{u}} + S_{T_0} \frac{T_0'}{\bar{T_0}} \quad (6)$$

where S_ρ, S_u and S_{T_0} are the sensitivities to density, velocity and total temperature respectively, defined as follows:

$$S_\rho = \frac{\partial logE_b}{\partial log\rho}_{u,T_0,\alpha,\beta=ct} \quad (7)$$

$$S_u = \frac{\partial logE_b}{\partial logu}_{\rho,T_0,\alpha,\beta=ct} \quad (8)$$

$$S_{T_0} = \frac{\partial logE_b}{\partial logT_0}_{\rho,u,\alpha,\beta=ct} \quad (9)$$

The sensitivities can be expressed in terms of non-dimensional parameters [6]:

$$S_\rho = \frac{1}{2}\left(\frac{\partial logNu}{\partial logRe} + \frac{\partial log\eta}{\partial logRe}\right) \quad (10)$$

$$S_u = S_\rho + \frac{1}{2m}\left(\frac{\partial logNu}{\partial logM} - \frac{1}{\tau_W}\frac{\partial log\eta}{\partial logM}\right) \quad (11)$$

$$S_{T_0} = \frac{1}{2}\left[n_t + 1 - m_t\frac{\partial logNu}{\partial logRe} - \frac{\theta - \eta}{\theta} + \frac{1}{\tau_w}\left(-\frac{1}{2m}\frac{\partial log\eta}{\partial logM} + m_t\frac{\partial log\eta}{\partial logRe}\right) - \frac{1}{2m}\frac{\partial logNu}{\partial logM}\right] \quad (12)$$

where $n_t = \partial logk/\partial logT_0$, $m_t = \partial log\mu/\partial logT_0$, $m = 1/(1 + M^2\,(\gamma - 1)/2)$.

In incompressible and supersonic flows $S_u = S_\rho = S_{\rho u}$, since the Mach number does not affect the heat transfer process ($Nu \neq f(M)$). Then the HW is sensitive to mass flux and total temperature fluctuations and eq. (6) can be simplified to:

$$\frac{e'}{\bar{E}} = S_{\rho u}\frac{(\rho u)'}{\overline{\rho u}} + S_{T_0}\frac{T_0'}{\bar{T_0}} \quad (13)$$

In high subsonic/transonic and subsonic slip flows though, there is a strong Mach (or Knudsen) effect on the heat transfer process, and $S_\rho \neq S_u$. In this case, eq. (6) can only be solved instantaneously by using at least 3 wires with sufficiently different sensitivities (e.g. different overheats) to solve a system with 3 eq. and 3 unknowns.

In literature there are discrepancies concerning the behavior of sensitivities. Horstman & Rose [11] found that for high overheat ratios ($\tau_w > 0.8$) and high Reynolds numbers ($Re_d > 20$), $S_u = S_\rho$. Nevertheless this was not supported by other studies, where the sensitivity to density was always higher than the sensitivity to velocity [12, 13].

The yaw angular sensitivity S_a for inclined wires in a planar flow field was first introduced by Motallebi [14] and expressed in the following form:

$$S_a = \frac{1}{2}\left(\frac{1}{\tau_w}\frac{\partial log\eta}{\partial \alpha} - \frac{\partial logNu}{\partial a}\right) \quad (14)$$

for three or more inclined wires in a three dimensional flow field, the pitch angle sensitivity S_β should also be included:

$$S_\beta = \frac{1}{2}\left(\frac{1}{\tau_w}\frac{\partial log\eta}{\partial \beta} - \frac{\partial logNu}{\partial \beta}\right) \quad (15)$$

and the sensitivity equation takes the following form:

$$\frac{e'}{\bar{E}} = S_\rho \frac{\rho'}{\bar{\rho}} + S_u \frac{u'}{\bar{u}} + S_{T_0} \frac{T_0'}{\bar{T_0}} + S_a a' + S_\beta \beta' \quad (16)$$

The fluctuating angles a' and β' can be computed by an angular calibration. Cukurel et al. [4] found that the angular calibration was independent of Mach and Reynolds number in the tested range.

The sensitivities can be directly obtained through a systematic calibration, where one parameter is varied while keeping the other two constant. This method requires extensive, lengthy calibrations in closed loop facilities.

METHODOLOGY

The proposed methodology for the calculation of sensitivities is based on the data reduction method for x-wire probes developed by Cukurel et al. [4]. This method consists in the use of an effective wire temperature and of empirical correlations to eliminate the dependency of the calibration on total temperature and Mach number of the flow respectively, which allows establishing a unique Nu-Re calibration curve. The procedure will be briefly described here, as more details can be found in Ref. [4].

The Nusselt number is computed by eq. (5), while the Reynolds number is defined by:

$$Re = \frac{\rho u d_w}{\mu} \quad (17)$$

The viscosity μ and the thermal conductivity k of the flow are evaluated on the total temperature of the flow.

The wire temperature is defined as the temperature which collapses a set of Nu-Re data obtained at different flow T_0 on a single curve. As in incompressible continuum flows there is no compressibility effect, the Nusselt number is a unique function of Reynolds. The R^2 of a 4th order polynomial fit is used as the selection criterion. The wire temperature defined this way is the temperature that better represents the convective heat transfer process from the wire to the flow. In Figure 1, a set of data obtained at different flow total temperatures is presented. When plotted in terms of voltage and velocity, a different curve is created for each temperature level. But when plotted in terms of Nu_w and Re_w with the effective wire temperature in eq. (5), all the data collapse in a single curve.

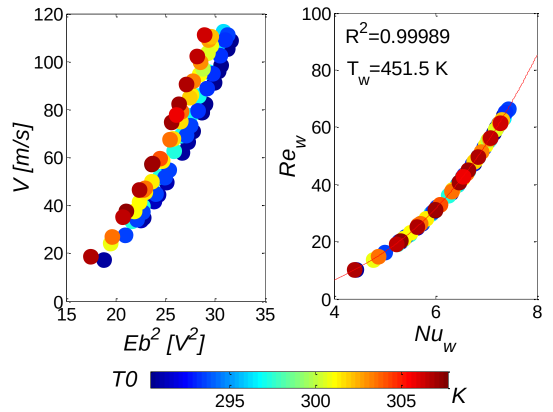

Figure 1: Low speed calibration to determine the wire temperature

In the compressible region, a correlation proposed by Dewey [9] is employed to eliminate the Mach number dependency:

$$Nu_{corr}(Re,\infty) = \frac{Nu(Re,M)}{\varphi(Re,M)} \tag{18}$$

The correction term $\varphi(Re,M)$ is used to compute a corrected Nusselt number which is only dependent on Reynolds. The correlation is valid for $M > 0.3$ and for all the Reynolds numbers. The correction is illustrated in Figure 2. Finally, a single $Nu_{w,corr} - Re_w$ calibration curve is obtained.

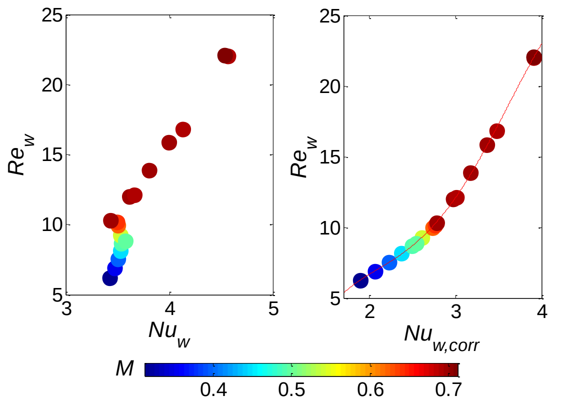

Figure 2: Nu-Re calibration with application of the Mach number correction

In order to compute the sensitivities, the set of equations 13-15 are used. The logarithmic derivatives of the recovery factor to Re and Mach $\left(\frac{\partial log\eta}{\partial logRe}, \frac{\partial log\eta}{\partial logM}\right)$ can be analytically computed by the empirical correlation proposed by Dewey [9]. The logarithmic derivatives of Nusselt to Re and Mach can be transformed to:

$$\frac{\partial logNu}{\partial logRe} = \frac{Re}{\varphi}\frac{\partial\varphi}{\partial Re} + \frac{\partial logNu_{corr}}{\partial logRe} \tag{19}$$

$$\frac{\partial logNu}{\partial logM} = \frac{M}{\varphi}\frac{\partial\varphi}{\partial M} \tag{20}$$

The terms $\frac{\partial\varphi}{\partial Re}$ and $\frac{\partial\varphi}{\partial M}$ are the partial derivatives of the correction term $\varphi(Re,M)$. The only term depending on the wire properties is $\frac{\partial logNu_{corr}}{\partial logRe}$, which can be directly obtained by the calibration.

APPLICATION

In this section, the methodology explained above is applied to the data obtained with three hot wire probes tested at different flow regimes. All probes feature tungsten wires of 9 μm diameter. A 3-wire probe with inclined wires is tested in incompressible flow, while two single-wire probes placed normal to the flow are tested in compressible flow. Angular effects will be thus discussed only for the former case. Finally, the sensitivities are compared with experimentally defined values given by Nagabushana & Stainback in Ref. [13].

Case 1: Incompressible continuum flow including angular effects

A hot wire probe with 3 slanted wires operated at different overheat ratios was tested in incompressible continuum flow. The flow conditions and wire characteristics are given in Table 1.

$d_{w1,2,3}$	9 μm
T_{w1}	490.75 K
T_{w2}	462.25 K
T_{w3}	430.75 K
$\boldsymbol{M}$	**0.01-0.26**
Re_w	5-44
$\boldsymbol{Kn_w}$	**0.0071-0.0079**
ρ	1.117-1.211 kg/m^3
V	1-90 m/s
T_0	291-315 K

Table 1: Hot wire characteristics and test flow conditions for Case 1.

In this case, according to the theory, there is no Mach number effect on the heat transfer process and the sensitivity to velocity is equal to the sensitivity to density, $S_u = S_\rho$. Calculated by eq. (10), the sensitivity to velocity for all three wires seems to be almost independent of the Reynolds and Mach numbers of the flow, as can be seen in Figure 3. Moreover, it is only slightly affected by the overheat, which results in small differentiation of its value for the three wires. More specifically, the sensitivity to velocity is almost equal for wires 1 and 3 ($S_{u1} = 0.161$,$S_{u3} = 0.1611$), despite having the biggest difference in overheat. A higher sensitivity ($S_{u2} = 0.1662$) is computed for wire 2, although it is operated at an intermediate temperature.

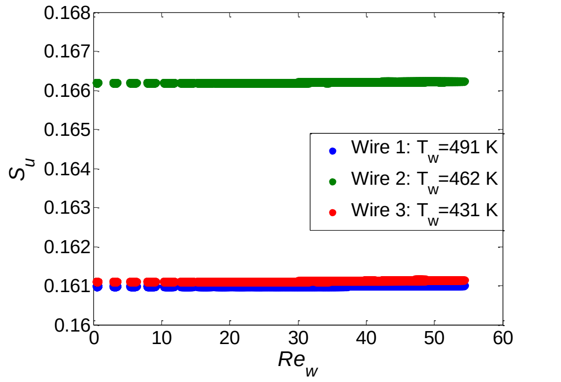

Figure 3: Sensitivity to velocity as a function of Reynolds number for all three wires.

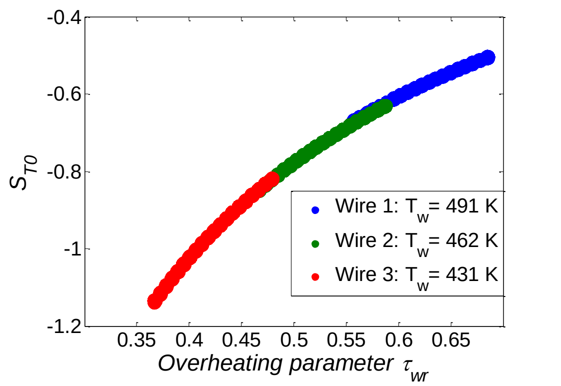

Figure 4: Total temperature sensitivity as a function of the overheating parameter for the three wires

In this range of conditions, the total temperature sensitivities seem to be independent of the Mach and Reynolds numbers and a unique function of the temperature difference between the heated wire and the flow. This is clearly illustrated in Figure 4, where the sensitivities of the three wires are plotted as a function of the overheating parameter τ_{wr}. The sensitivities are always negative, as an increase of the flow temperature results in a decrease of the heat transfer rate. As the overheat increases, the wires are rendered less sensitive to total temperature fluctuations. It should be noted that the absolute value of the total temperature sensitivity is significantly higher to that of the velocity sensitivities, even for the highest overheat. Hence, the assumption of negligible total temperature fluctuations should be treated with caution in such conditions, as it could lead to errors.

Since $S_u = S_\rho$ and the hot wire is sensitive to all three velocity components, the sensitivity equation for this case is written as:

$$\frac{e'}{\overline{E}_i} - S_{a,i}a' - S_{\beta,i}\beta' = S_{\rho u,i}\frac{(\rho u)'}{\overline{\rho u}} + S_{T_{0i}}\frac{T_0'}{\overline{T_0}} \qquad (21)$$

for i=1, 2, 3.

The sensitivities to the flow angles S_a and S_β are a function of the flow Reynolds number and of the flow angles. In Figure 5, they are presented as a function of the angle for a range of ±30°, where $a, \beta = 0°$ corresponds to the case where the flow is aligned to the probe body. The sensitivity of the wires to the yaw and pitch angle depends on their orientation. It can be seen that wire 1 is more sensitive to the pitch angle, while wire 2 is more sensitive to the yaw angle.

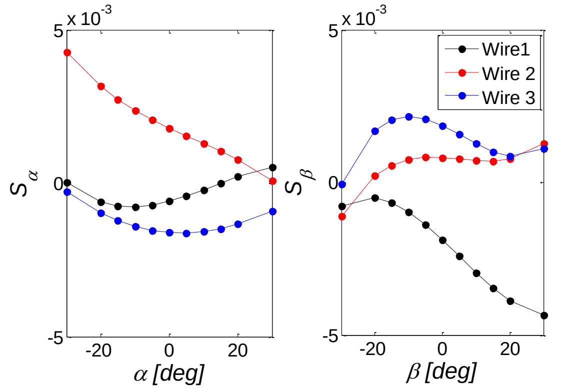

Figure 5: Angular sensitivities. Left: Yaw sensitivity as a function of the yaw angle for $\beta = 0°$ and Re=8.5. Right: Pitch sensitivity as a function of the pitch angle for $\alpha = 0°$ and Re=8.5.

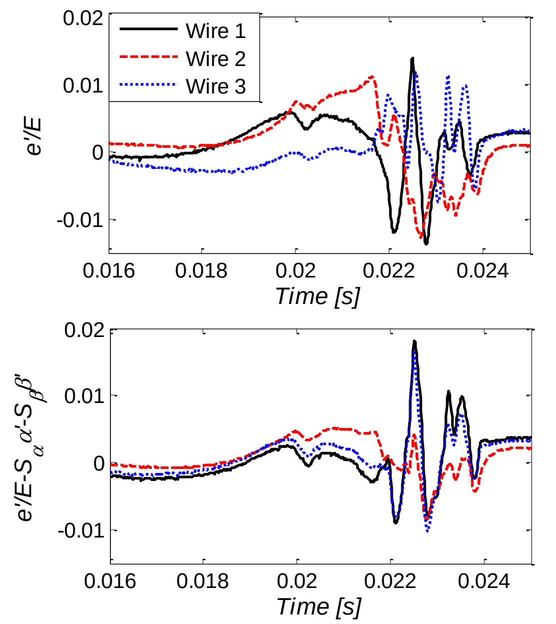

Figure 6: Time-resolved voltage and angular compensation.

The fluctuating angles can be directly obtained by an angular calibration, so the angular terms are considered known and are moved to the left hand side of Eq. (21). Subtracting the angular terms from the voltage term in Eq. (21), results in the correlation of the voltages of the three wires, as can be seen in Figure 6. This behavior has been observed by Cukurel et al. [4] for an X-wire probe, only sensitive to the yaw angle. By compensating for the angular sensitivity, the problem of the 3 slanted wires is equivalent to that of 3 parallel wires normal to the flow. Thus only two unknowns remain ($(\rho u)'/\overline{\rho u}$) and ($T_0'/\overline{T}_0$) and the system can be solved by employing any set of two wires. It is reasonable in this case to choose the two wires with the highest and lowest overheat, in order to maximize the difference in the total temperature sensitivity values.

Case 2: High subsonic slip flow

This Hot Wire was tested in high subsonic ($M = 0.605 - 0.67$) slip flow ($Kn = 0.0567 - 0.1139$**)**. The hot wire characteristics and test flow conditions are presented in Table 2. In this range, the sensitivities to density and velocity are expected to be different and three wires with different sensitivity values will be needed to solve the sensitivity equation system.

d_w	9 μm
T_w	475 K
M	**0.605-0.67**
Re_w	8.06-17.3156
Kn_w	**0.0567-0.1139**
ρ	0.0806-0.1629 kg/m^3
V	197-216 m/s
T_0	289-295 K

Table 2: Hot-wire characteristics and test flow conditions for Case 2.

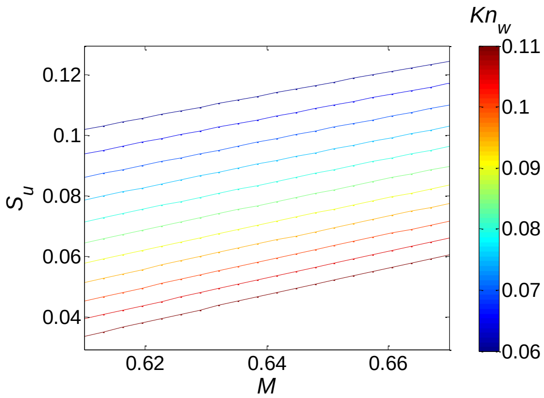

Figure 7: Sensitivity to velocity as a function of Mach and Knudsen numbers.

The sensitivities to velocity and density are presented in Figure 8. They are plotted in terms of Mach and Knudsen numbers in order to highlight the gas rarefaction effects. High Knudsen numbers correspond to low densities. Both sensitivities are highly dependent on Knudsen and Mach, even for the narrow Mach number range that was tested in this case. They both increase with increasing Mach number and decrease with increasing Knudsen number. The sensitivity to density is always higher than the sensitivity to velocity, with the difference being amplified for the higher Knudsen numbers (lower densities), as can be seen in Figure 9. The sensitivities to density and velocity are not affected by the small total temperature variations that were present in this case.

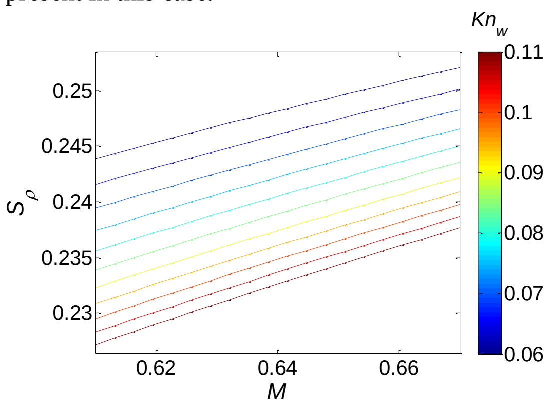

Figure 8: Sensitivity to density as a function of Mach and Knudsen numbers

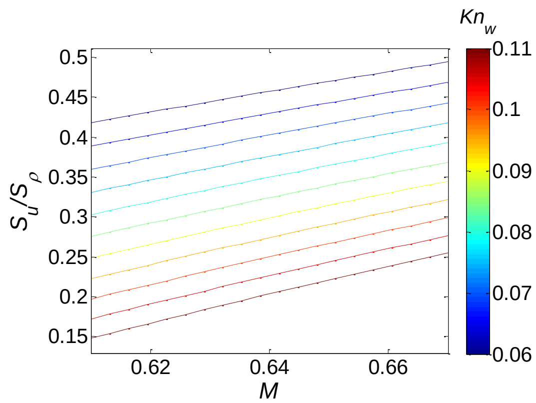

Figure 9: Sensitivity to velocity over sensitivity to density as a function of Mach and Knudsen numbers

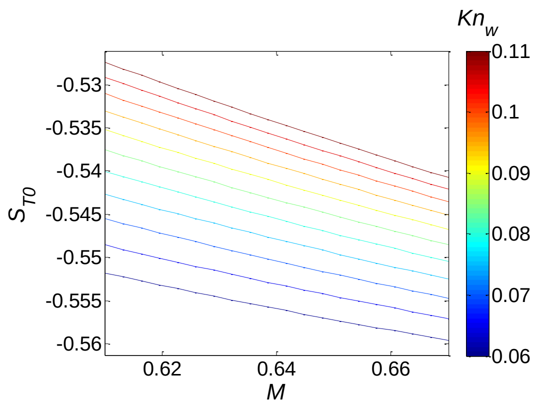

Figure 10: Sensitivity to total temperature as a function of Mach and Knudsen numbers for T0=292 K.

The total temperature sensitivity is a function of the Mach, Knudsen and total temperature of the flow. Its absolute value increases with increasing

Mach and decreasing Reynolds numbers as can be seen in Figure 10. As observed in the previous section, the total temperature sensitivity is increasing with increasing total flow temperature, due to the decrease of overheat. This behavior is illustrated in Figure 11.

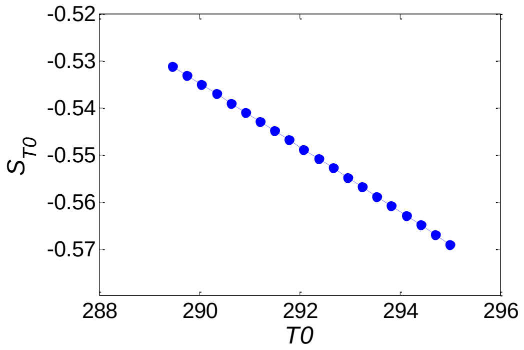

Figure 11: Sensitivity to total temperature as a function of total temperature for M=0.61 and Kn_w=0.075.

Case 3: Low subsonic to high subsonic continuum flow

In this case, a hot wire was tested in continuum flow ($Kn_w < 0.01$) in a Mach number range from 0.1 to 0.55. The wire characteristics and the flow conditions are summarized in Table 3.

d_w	9 μm
T_w	459 K
M	**0.1-0.55**
Re_w	22.6-111.5
Kn_w	**0.007-0.0072**
ρ	1.2-1.2308 kg/m^3
V	36-180 m/s
T_0	280-287 K

Table 3: Case 3: Hot-wire characteristics and test flow conditions

As previously explained, to correct for the Mach number dependency on the heat transfer, a correlation proposed by Dewey [9] is employed that results in a corrected Nusselt, independent of Mach and a unique function of Reynolds. This correlation is valid for M>0.3, which results in two different curves, as can be seen in the left of Figure 12:

- Curve A: For M>0.3, Dewey's correlation is applied
- Curve B: M<0.3, no correction applied, $Nu = Nucorr$.

This behavior can create difficulties when measuring in Mach number's around 0.3. It is preferable to have a single calibration curve covering all the Mach number range. For this reason, a correlation proposed by Klopfer [15] is employed, which constitutes an extension to Dewey's correction to cover the range $0 < M < 0.4$. Klopfer uses a weighted logarithmic average between the corrected Nusselt number at $M = 0.4$ by Dewey and the Oseen solution at $M = 0$ by Cole and Roshko [16]. This extension was validated on a set of experimental data ranging over $0.1 < Re < 300$. In the right part of Figure 12, the extended correlation is applied, creating a continuous calibration curve for the whole Mach number range:

- For $M \geq 0.4$: Dewey's correlation
- For $M < 0.4$: Klopfer's extension

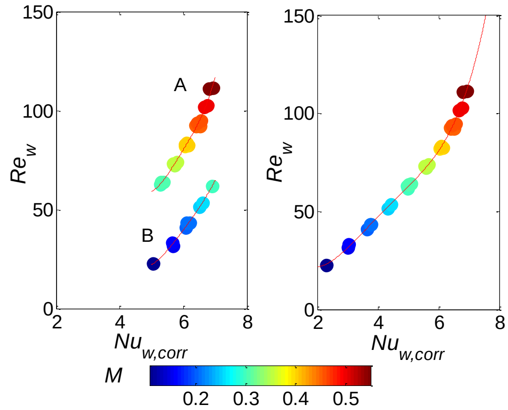

Figure 12: Left: Application of Dewey's correlation, Right: Addition of Klopfer's extension.

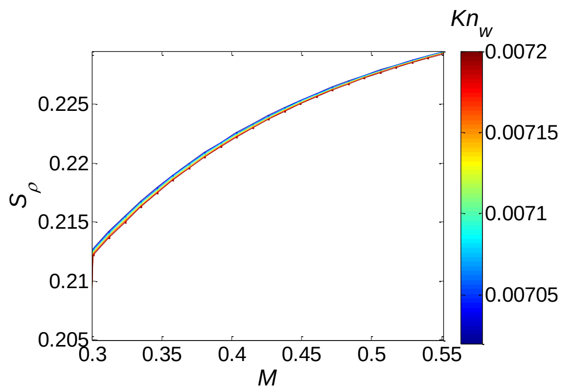

Figure 13: Sensitivity to density as a function of Mach and Knudsen numbers.

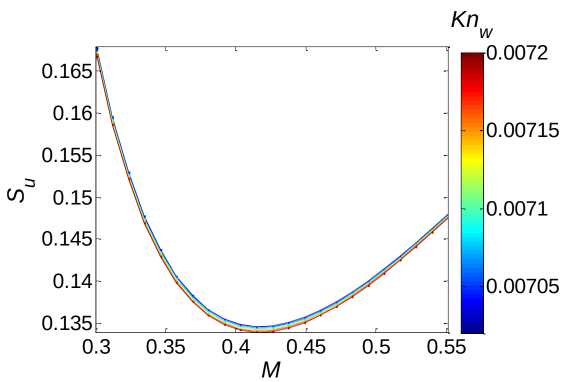

Figure 14: Sensitivity to velocity as a function of Mach and Knudsen numbers.

The sensitivities are presented in Figures 13-17. Klopfer's extension doesn't provide reasonable results, so only the data for $M > 0.3$ are presented, where Dewey's correlation is applied. Nevertheless, Klopfer's extension can still be useful to acquire the Reynold's number of the flow. The sensitivity to density presents similar behavior with case 2: it increases with increasing Mach number. It also increases slightly with decreasing Knudsen number: this effect is very small for all sensitivities, since there is only a small variation of density for this case.

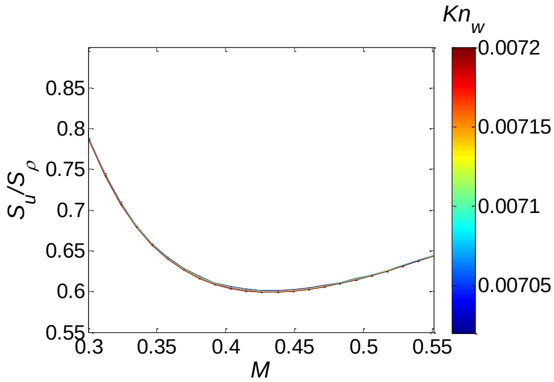

Figure 15: Sensitivity to velocity over sensitivity to density as a function of Mach and Knudsen numbers

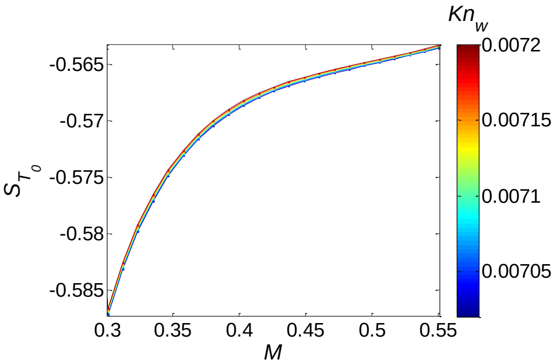

Figure 16: Sensitivity to total temperature as function of Mach and Knudsen numbers, for T0=282 K.

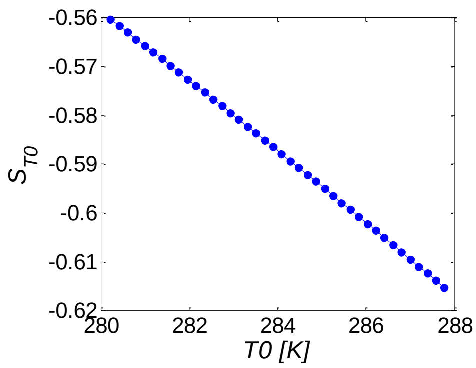

Figure 17: Sensitivity to total temperature as a function of total temperature, for M=0.4 and Kn_w=0.0071.

The sensitivity to velocity increases with the Mach number, as in case 2, for $M > 0.42$. But a sharp increase is observed towards lower Mach numbers. This behavior is caused by the term $\partial logNu/\partial logM$ (Eq. (11)) which is computed by Eq. (20), so is solely based on the correlation. This effect can be seen also in the behavior of the sensitivity to total temperature in Figure 16, which decreases rapidly for $M > 0.42$. It should be noted that even for higher Mach numbers its behavior is opposite to that observed in case 2. But as also observed in the previous sections, it is a strong function of temperature, increasing with increasing total flow temperature. Finally, as it can be seen in Figure 15, the sensitivity to velocity is always lower than the one of density, even at the proximity of the incompressibility limit of $M = 0.3$.

Comparison with literature

In this section, the data presented before are compared with the data of Nagabushana and Stainback given in Ref. [13]. It is a collection of experimental data obtained in subsonic slip flow conditions with CTA anemometers, where the sensitivities to temperature, velocity and density are computed with two different methods. The results that will be included here were obtained with a single wire placed normal to the flow, in the conditions presented in Table 4. The sensitivities presented were computed based on the functional dependency of the voltage to velocity, density and total temperature $E_b = f(\rho, u, T_0)$ and not by using non-dimensional values. The total temperature sensitivity data of Nagabushana & Stainback have significant scatter and no observations could be made. Hence, they would not be included in the following comparison.

d_w	3.8 μm
T_w	583 K
M	**0.05-0.49**
Re_w	2-132
Kn_w	**0.0022-0.0388**
ρ	0.57-9.5 kg/m^3
V	17.4-122 m/s
T_0	299-324 K

Table 4: Wire characteristics and flow conditions, Stainback & Nagabushana [13].

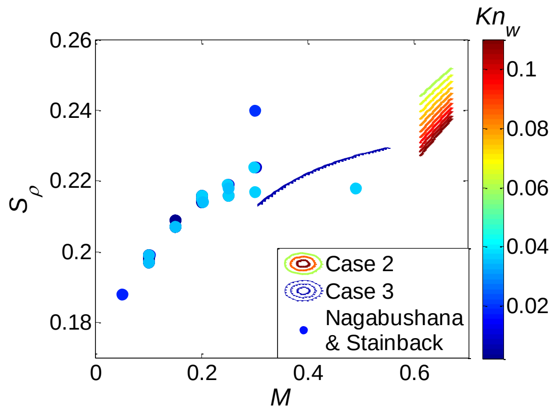

Figure 18: Sensitivity to density.

The sensitivities to density for Cases 2 and 3 are plotted together with the ones of Nagabushana & Stainback [13] in Figure 18. For all cases, the sensitivity is increasing with increasing Mach number. In the case of Nagabushana & Stainback, there seems to be no effect of the Knudsen number, contrary to cases 2 and 3. Nevertheless, for Case 3 the Knudsen number varies in a very small range, while Case 2 data are obtained at higher Mach and Knudsen numbers. Thus, further investigation is required before a conclusion can be reached. It can be seen though that the values of case 3 are close to the values obtained by Stainback & Nagabushana. .

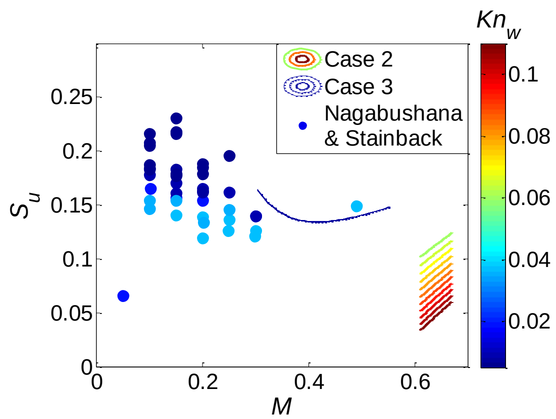

Figure 19: Sensitivity to velocity.

The sensitivities to velocity are presented in Figure 19. The data of Nagabushana & Stainback feature significant scatter and it is difficult to discern clear patterns. Nevertheless, it seems that the sensitivity decreases for increasing Knudsen numbers (lower densities) as is also observed for case 3. Moreover, there seems to be a decreasing trend with increasing Mach number which matches the behavior of Case 3 for $M > 0.42$. Again, more data are required in order to draw conclusions and be able to evaluate the behavior of the velocity sensitivity

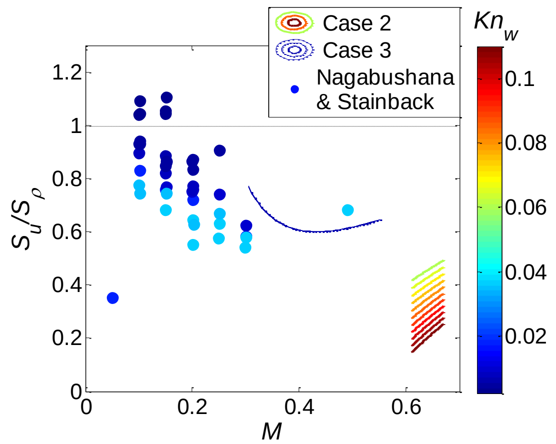

Figure 20: Sensitivity to velocity over sensitivity to density

In Figure 20, the ratios of the sensitivity to velocity over the sensitivity to density are presented. Except for a few points of Nagabushana & Stainback, the sensitivity to velocity is always lower than the sensitivity to density. With the correlation in use, the sensitivity to velocity can never be higher than the sensitivity to density for $M > 0.3$, as the term added to S_ρ in Eq. (11) is always negative. The ratio decreases with increasing Knudsen (lower velocities) for all cases, implying that at low density flows a hot wire is significantly more sensitive to density than to velocity. Thus, this should be considered when measuring in such kind of flows.

CONCLUSIONS

The HWA sensitivities to density, velocity and total temperature have been studied in different flow conditions, computed with a data reduction methodology, which employs an empirical correlation to evaluate the compressibility effect on the heat transfer from the wire to the flow. An extension to the correlation was tested to cover the Mach number range for M<0.3, but did not provide reasonable sensitivity values. Thus, the possible compressibility effect due to gas rarefaction for $M < 0.3$ could not be considered.

For all the cases tested, it is clear that the sensitivity to total temperature is mainly dependent on the overheat, while slightly affected by Knudsen and Mach. Thus, by varying the temperature of the wire, very different sensitivities can be achieved. On the contrary, the overheat seems to have a small influence on the sensitivities to density and velocity, even when the wire temperature is significantly varied (Case 1). As a result, using multiple wires operated at different temperatures might not be adequate in order to solve the sensitivity equation system. Other solutions should be investigated, like using different wire diameters (different Knudsen numbers), since the Knudsen number seems to have an important effect, at least on the velocity sensitivity.

The absolute value of the total temperature sensitivity is always more than 50% higher than the ones of velocity and density. The common assumption of negligible total temperature fluctuations should thus be treated with caution for each application, unless much higher overheats are used. In addition, the sensitivity to density is significantly higher than the sensitivity to velocity for low density flows, a fact that should be taken into account in such applications.

In order to evaluate the methodology, the results are compared with experimentally computed sensitivities by Nagabushana & Stainback [13]. Unfortunately these data cover a small range of conditions and there is limited overlap with the current cases. Nevertheless, similar trends are observed and reasonable sensitivity values. In any case, further investigations are required before the applicability of this methodology is assessed.

To conclude, the most important remarks concerning the application of HWA are repeated here:

- When using typical overheating parameters of 0.7-0.8, S_{T0} is more than 50% higher than S_ρ and S_u. Hence, total temperature fluctuations can have an important effect on the measurement.
- In slip flows S_ρ is significantly higher than S_u, so density fluctuations can influence the measurement.
- Different overheats do not suffice to differentiate S_ρ and S_u in order to decouple the velocity and density fluctuations using the sensitivity equation. Other solutions should be investigated.

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