MULTI-HOTWIRE PROBE SENSITIVITY OPTIMIZATION IN CONSTANT TEMPERATURE ANEMOMETRY (CTA) FOR TRANSONIC FLOWS

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ABSTRACT

A simple optimization method is proposed for multi-wire CTA probe design. The optimization computes the sensitivities to density, velocity and total temperature for each wire, based on empirical $Nu - M - Re$ and $\eta - M - Re$ correlation for heated cylinders in compressible flow. This correlation is a reasonable estimator for the compressibility effect, enabling the reduction to M-independent relation. The output is a set of optimal wire temperatures and diameters, recommended for probe production. Following the manufacturing, it is necessary to calibrate each wire for $Re - Nu$ relationship and apply together with the empirical compressibility relation. The technique allows the use of a single probe with multiple wires across a range of transonic flows with various conditions.

NOMENCLATURE

α	Angle between the measured voltages vector $\underline{E}_{me}$ and the vector $\underline{A} \cdot \underline{F}$
ζ	Measure of how much $\left\|\|\underline{A} \cdot \underline{F}\|\right\|$ falls short of its maximum possible value
η	Recovery factor, T_r/T_0
θ	Overheat ratio T_w/T_0
$\kappa(\underline{A})$	Condition number of matrix $\underline{A}$
ρ	Freestream Flow density
σ^2	Variance of condition number with respect to different flow combinations
τ_{wr}	Overheating parameter, $(T_w - T_r)/T_r$
Φ	Compressibility correction function
$\underline{A}$	Probe sensitivity matrix
d_w	Wire diameter
E	wires voltages
$\underline{E}_{me}$	Vector of measured normalized wires voltages perturbations
$\underline{F}$	Vector of decoupled normalized flow perturbations
h	Convection heat transfer coeff
k_f	Thermal conductivity of the fluid
l	Wire length
M	Freestream Mach number
m	$[1 + (\gamma - 1)/2 \cdot M^2]^{-1}$
m_t	$\partial log\, \mu / \partial log T_0$
n_t	$\partial log k_f / \partial log T_0$
Nu	Nusselt number
Q_w	Convective heat transfer rate from a wire
Re	Reynolds number
Re_{T_0}	Reynolds number based on wire diameter, with viscosity evaluated at T_0
S_u	Sensitivity to velocity perturbation
S_ρ	Sensitivity to density perturbation
S_{T_0}	Sensitivity to total temperature perturb
s_{max}, s_{min}	Largest and smallest singular values of $\underline{A}$
T_0	Freestream flow total temperature
T_w	Wire temperature
ΔT_{pert}	Temperature perturbation applied for estimate of sensitivity matrix perturb. ($\underline{\Delta A}$)
u	Freestream velocity
$\underline{(\cdot)'}$	Perturbation of a quantity
$\overline{(\cdot)}$	Mean of a quantity
$(\cdot)_i$	index of flow combination i
$\|\|\cdot\|\|$	Frobenius norm
Δ	Small change in quantity

INTRODUCTION

Constant temperature anemometry (CTA) measurements for transonic flow conditions are typically very demanding, as there are simultaneous perturbations of velocity, density and total temperatures. Therefore, defining the attributes of a probe to operate under these conditions is non-trivial.

Numerous probe design guidelines suggest good working practices towards admissible ranges of wire length to diameter ratio, wire material and coating, prong geometry and system damping [1].

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Moreover, for low speed applications, guidelines relating the system frequency response to wire diameter and overheat ratio exist. In contrast, for transonic applications, where the wire diameter and its temperature are critical factors, guidelines are limited. In transonic flow regimes, the selection process of wire diameter and its temperature should be driven by density, velocity and temperature sensitivities. Therefore, a-priori estimation of the probe sensitivities would be useful.

Historically, by independently varying the flow quantities in the transonic region, a direct method empirically determines the sensitivities of each wire [2]. This is usually accompanied by uncertainties and the ill-posing of the sensitivities matrix, requiring over-determined methods to reduce noise levels [3]. Sometimes a simplification of the sensitivities can be used for high overheat ratios [4].

In theory, if heat losses which are not associated with convection (radiation, conduction losses) are well-known, and the probe geometry including the wire thickness is well-determined, there exists a relation which characterizes the convective heat transfer from the hot wire [5]. However, in reality, the contribution of these unknowns to the system response negate the possibility of creating a universal calibration.

Conventionally, the well-accepted relation of heat transfer over a specific wire is given by [6],

$$Nu = \frac{hd_w}{k_f} = f(Re_{T_0}, M, \theta), \quad (1)$$

where Re_{T_0} is the Reynolds number based on wire diameter with viscosity evaluated at T_0, and the overheat ratio is $\theta = \frac{T_w}{T_0}$.

Considering that convective heat transfer rate from the wire is

$$Q_w = \pi d_w l h(T_w - \eta T_0), \quad (2)$$

it is implied that h is invariant for small changes (~<100K) in surface or gas temperature. However, Nu dependence on thermal loading has been reported. But, for moderately transonic flows ($0.5 < M < 0.8$) and temperature differentials higher than $(T_w - T_0) > \sim 70K$, it can be neglected [7]. Furthermore, for small T_0 perturbations, which result in small $(T_w - T_0)$ variations, the change in Nu is insignificant across all M.

Therefore, it can be a good practical approximation to assume the reduction of the overheat dependency,

$$Nu = f(Re_{T_0}, M). \quad (3)$$

This representation is consistent with scientific community on slip flows over heated wires [8],[9]. The applicability of this assumption to transonic hotwire anemometry has been demonstrated by [10].

METHODOLOGY

1. Wire sensitivity evaluation:

Sensitivity based, compressible flow turbulence measurements can be described by the individual contribution of the velocity, density, and T_0 perturbations to the voltage perturbation relative to the mean values:

$$\frac{E'}{\overline{E}} = S_u \cdot \frac{u'}{\overline{u}} + S_\rho \cdot \frac{\rho'}{\overline{\rho}} + S_{T_0} \cdot \frac{T_0'}{\overline{T_0}}, \quad (4)$$

where E is the wire voltage, $(\cdot)'$ is for perturbation, $\overline{(\cdot)}$ is for mean quantities. The sensitivities to velocity, density and total temperature perturbations S_u, S_ρ, S_{T_0} are defined as:

$$S_u(\rho, u, T_0) = \left.\frac{\partial logE}{\partial logu}\right|_{\rho, T_0 = const}, \quad (5)$$

$$S_\rho(\rho, u, T_0) = \left.\frac{\partial logE}{\partial log\rho}\right|_{u, T_0 = const}, \quad (6)$$

$$S_{T_0}(\rho, u, T_0) = \left.\frac{\partial logE}{\partial logT_0}\right|_{u, \rho = const}. \quad (7)$$

For CTA, the derivation of analytical sensitivities, utilizing logarithmic derivatives of $Nu - Re - M$ and $\eta - M - Re$ relationships, are portrayed in [6]:

$$S_\rho = 0.5\left(\frac{\partial logNu}{\partial logRe_{T_0}} - \frac{1}{\tau_{wr}}\frac{\partial log\eta}{\partial logRe_{T_0}}\right), \quad (8)$$

$$S_u = S_\rho + \frac{1}{2m}\left(\frac{\partial logNu}{\partial logM} - \frac{1}{\tau_{wr}}\frac{\partial log\,\eta}{\partial logM}\right), \quad (9)$$

$$S_{T_0} = 0.5\left[n_t + 1 - m_t\frac{\partial logNu}{\partial logRe_{T_0}} - \frac{\theta}{\theta - \eta} + \frac{1}{\tau_{wr}}\left(\frac{1}{2m}\frac{\partial log\,\eta}{\partial logM} + m_t\frac{\partial log\,\eta}{\partial logRe_{T_0}}\right) - \frac{1}{2m}\frac{\partial logNu}{\partial logM}\right]. \quad (10)$$

An example of a convenient semi-empirical formulation for infinite length wires is provided in [8],

$$Nu(Re_{T_0}, M) = Nu(Re_{T_0}, \infty) \cdot \Phi(Re_{T_0}, M). \quad (11)$$

This formulation states in a disjoint manner the Mach independent behavior of Nu, $Nu(Re_{T_0}, \infty)$, and the compressibility correction, $\Phi(Re_{T_0}, M)$.

For convenience, the empiric relations are repeated here:

$$Nu(Re_{T_0}, \infty) = Re_{T_0}^{n}\left[0.14 + 0.2302 \cdot \left(\frac{Re_{T_0}^{0.7114}}{15.44 + Re_{T_0}^{0.7114}}\right) + \left(\frac{0.01596}{0.3077 + Re_{T_0}^{0.7378}}\right) \cdot \left(\frac{15}{15 + Re_{T_0}^{3}}\right)\right], \quad (12)$$

$$n = 1 - 0.5 \cdot \left(\frac{Re_{T_0}^{0.6713}}{2.571 + Re_{T_0}^{0.6713}}\right), \quad (13)$$

$$\Phi(Re_{T_0}, M) = 1 + A(M) \cdot \left[1.834 - 1.634\left(\frac{Re_{T_0}^{1.109}}{2.765 + Re_{T_0}^{1.109}}\right)\right] \cdot \left[1 + \left(0.3 - \frac{0.065}{M^{1.67}}\right) \cdot \left(\frac{Re_{T_0}}{4 + Re_{T_0}}\right)\right], \quad (14)$$

$$A(M) = \frac{0.6039}{M} + 0.5701\left[\left(\frac{M^{1.222}}{1 + M^{1.222}}\right)^{1.569} - 1\right]. \quad (15)$$

Thus, implementing the assumption of Eq (3), along with the logarithmic derivation of Nu and η relations from Ref. [8], estimation of individual wire sensitivities is possible. This is inline with the hypothesis of universal calibration procedure described in [11]. Therefore, this solution can be used towards an optimization of the multi-wire probe sensitivity matrix for either a specific or a broad range of flow conditions.

Generally, the use of the universal relations mentioned above must be applied with caution. The validity of the correlations is only for infinitely long wires with defined exact diameters. Accounting for the end loss correction improves the sensitivity estimate [12][13], however wire-specific calibration is still necessary for a reasonable accuracy. Therefore, it is advisable to empirically calibrate each probe before use and not rely on estimated sensitivities. On the other hand, it was demonstrated that the compressibility correction Φ and the recovery η correlation can still be applied to real, finite wires [10].

In order to characterize the trends associated with selected wire diameters and temperatures, as a first order approximation, the use of full empirical relations is sufficient. Therefore, this investigation utilizes the formulations outlined in Ref. [8].

2. Probe sensitivity matrix

For compressible transonic flow with work edition (common in turbomachinery applications), perturbations in local velocity, density and stagnation temperature are expected. In order to deduce the measured voltage fluctuations into different flow perturbations, a minimum of 3 wires with different sensitivities are required. Then, each probe (based on the number of wires -3, 4, or more) will have its own sensitivity matrix.

For example, a sensitivity matrix $\underline{A}$ for a 4 wire probe would satisfy the following equation:

$$\underline{A} \cdot \underline{F} = \underline{E}_{me}, \tag{16}$$

where

$$\underline{A} = \begin{bmatrix} S_{u1} & S_{\rho 1} & S_{T_0 1} \\ S_{u2} & S_{\rho 2} & S_{T_0 2} \\ S_{u3} & S_{\rho 3} & S_{T_0 3} \\ S_{u4} & S_{\rho 4} & S_{T_0 4} \end{bmatrix} ;\ \underline{F} = \begin{pmatrix} \frac{u'}{\overline{u}} \\ \frac{\rho'}{\overline{\rho}} \\ \frac{T_0'}{\overline{T_0}} \end{pmatrix} ;\ \underline{E}_{me} = \begin{bmatrix} \frac{E_1'}{\overline{E}_1} \\ \frac{E_2'}{\overline{E}_2} \\ \frac{E_3'}{\overline{E}_3} \\ \frac{E_4'}{\overline{E}_4} \end{bmatrix} \tag{17}$$

Overdetermined systems as such can allow greater resolution of individual flow quantities (instantaneous density, velocity, and temperature), along with decreased noise amplification.

3. Selection of the cost function:

Based on a set of wire diameter and temperature constrains, the optimization method determines the optimal combination of wire properties in a given probe within the allowable range over a range of flow conditions for a given cost function. The choice of the cost-function for the optimization is arbitrary. There are no clear demands that can define a cost function in a unique manner.

The optimization evaluates the sensitivity matrix for all combinations of wire diameters and temperatures at different flow conditions. In following, according to the cost function selected, a "grade" is assigned to each probe based on all the calculated sensitivity matrixes stemming from various flow parameter combinations.

Towards formulating the cost function, the condition number of matrix $\kappa(\underline{A})$ can be a useful parameter. Since the ultimate goal is to decouple the separate perturbations by solving Eq. (16), the quality of the solution depends on the well-posedness of the sensitivity matrix $(\underline{A})$. Higher condition number indicates a larger manifestation of the error from the input wire-voltages vector in the output set of flow perturbations. There are a multitude of ways in computing the condition-number of a matrix; in this investigation, Singular Value Decomposition (SVD) is used,

$$\kappa(\underline{A}) = \frac{s_{max}}{s_{min}}, \tag{18}$$

where s_{max}, s_{min} correspond to the largest and smallest singular values. $\kappa(\underline{A})$ represents the accuracy of pseudo-inverting a non-square over-defined matrix via least square fitting.

Another relevant parameter may be the robustness of the matrix condition number across a wide range of flow conditions. Such robustness can be represented by the variance of the condition numbers of all matrixes related to a single probe:

$$\sigma^2 = \overline{(\kappa_i - \overline{\kappa_i})^2} \tag{19}$$

Here κ_i is a short notation for $\kappa(\underline{A}_i)$, index i indicates the different flow conditions evaluated.

Looking for other possible cost function objectives, it is possible to directly consider the voltage noise propogation in the solution vector $\underline{F}$, described in Ref. [14] at chapter 18,

$$\frac{\left\|\Delta \underline{F}\right\|}{\left\|\underline{F}\right\|} \le \frac{\kappa(\underline{A})}{\zeta \cdot cos(\alpha)} \frac{\left\|\Delta \underline{E}_{me}\right\|}{\left\|\underline{E}_{me}\right\|}, \tag{20}$$

where α is the angle between the measured voltages vector $\underline{E}_{me}$ and the vector $\underline{A} \cdot \underline{F}$ (range of $\underline{A}$). The deviation is due to the inability of the columns of $\underline{A}$ to map the entire domain of $\underline{E}_{me}$. Small α would result in a better fit of the decoupled flow perturbations $\underline{F}$. ζ is the measure of how much $\left\|\underline{A} \cdot \underline{F}\right\|$ falls short of its maximum possible value,

$$\zeta = \frac{\left\|\underline{A}\right\| \left\|\underline{F}\right\|}{\left\|\underline{A} \cdot \underline{F}\right\|} \ ;\ 1 \le \zeta \le \kappa(\underline{A}). \tag{21}$$

Alternatively, another source of noise arises from the inaccurate determination of sensitivity matrix $\underline{A}$ in an experimental setting. Typically, this would manifest itself from an error in the evaluated wire temperature; and therefore a probe, which is less sensitive to slight deviations in wire temperature, may be desirable. The errors of the solution vector $\underline{F}$ due to sensitivity matrix noise can be written as [14],

$$\frac{||\Delta \underline{F}||}{||\underline{F}||} \leq \left(\frac{\kappa(\underline{A})^2 \tan(\alpha)}{\zeta} + \kappa(\underline{A})\right)\frac{||\Delta \underline{A}||}{||\underline{A}||}, \qquad (22)$$

where $\Delta \underline{A}$ is the sensitivity matrix error estimate computed by the maximum of all possible wire temperature perturbation (ΔT_{pert}) combinations. More specifically, $\Delta T_{pert} = \pm 1K$ is considered; other values ($\pm 2K$ or $\pm 5K$) yield similar results. In this investigation, the matrix norm $||\underline{A}||$ and the perturbation norm $||\Delta \underline{A}||$ are calculated using the Frobenius norms.

Addressing the angle α and ζ is not in the scope of this paper, as additional analysis in artificial voltage signal simulation is required in order to minimize α and maximize ζ. Therefore, ignoring the contribution of α and ζ to the problem, the Eq. (20) and Eq. (22) reduce to,

$$\frac{||\Delta \underline{F}||}{||\underline{F}||} \leq \kappa(\underline{A})\frac{||\Delta \underline{E}_{me}||}{||\underline{E}_{me}||}, \qquad (23)$$

$$\frac{||\Delta \underline{F}||}{||\underline{F}||} \leq \kappa(\underline{A})\frac{||\Delta \underline{A}||}{||\underline{A}||}. \qquad (24)$$

Thence, in order to improve invertabillity and minimize the amplification of the errors into the decoupled flow perturbations, we are interested in minimizing $\kappa(\underline{A})\frac{||\Delta \underline{A}||}{||\underline{A}||}$ and $\kappa(\underline{A})\frac{||\Delta \underline{E}_{me}||}{||\underline{E}_{me}||}$. However, as the normalized error in voltage, $\frac{||\Delta \underline{E}_{me}||}{||\underline{E}_{me}||}$, can not be accurately predicted, then the latter objective reduces to minimizing $\kappa(\underline{A})$ alone.

From another perspective, we are interested in detecting small perturbations, and therefore need the sensitivity values to be larger; this translates into maximization of $||\underline{A}||$. This is inline with minimizing $\kappa(\underline{A})\frac{||\Delta \underline{A}||}{||\underline{A}||}$; however, $\kappa(\underline{A})$ is the dominant factor, and for a single-value objective function, minimization of $\kappa(\underline{A})\frac{||\Delta \underline{A}||}{||\underline{A}||}$ is drawn towards the smaller condition number, without any control of the matrix norm.

Thus, a value function of two objectives is proposed,

$$Val = max\left(min\left(A \cdot \frac{\overline{\left(\kappa(\underline{A})||\Delta \underline{A}||\right)}_{min}}{\overline{\kappa(\underline{A})||\Delta \underline{A}||}}, B \cdot \frac{\overline{||\underline{A}||}}{\overline{||\underline{A}||}_{max}}\right)\right), \qquad (25)$$

where $\overline{\left(\kappa(\underline{A})||\Delta \underline{A}||\right)}_{min}, \overline{||\underline{A}||}_{max}$ are the minimal and maximal values of the considered probe collection. With this normalization, the range of the optimization objectives individually span between 0 and 1. A max-min formulation is used in the case of convex Pareto front shape. This value function maximizes $\overline{||\underline{A}||}$ and $\frac{1}{\overline{\kappa(\underline{A})||\Delta \underline{A}||}}$, and thus minimizing $\frac{\overline{\kappa(\underline{A})||\Delta \underline{A}||}}{\overline{||\underline{A}||}}$ as desired. The weights A and B are the choice of the user and it should be noted that the larger weight draws the result to the other objective.

4. Input parameters:

In the current investigation, it is possible to conduct an optimization towards a robust probe that can perform reasonably well in a broad spectrum of mean flow conditions or an optimal probe for a very specific mean flow condition.

(a) Wide range of mean flow conditions:

The maximal stagnation pressure ratio of 1.7 is chosen considering a typical highly loaded fan stage in a turbomachinery propulsion application. The upper limit on the total flow temperature is calculated using the isentropic flow relations corresponding to the compression. The density is deduced from the ideal gas state equation. Encompassing a wide range of mean flow conditions, total of 60 combinations are examined from the following ranges:

$$\mathbf{0.5 \leq M \leq 0.9,}$$
$$\mathbf{290K \leq T_0 \leq 350K,}$$
$$\mathbf{1atm \leq P_0 \leq 1.7atm.}$$

(b) Specific mean flow condition:

In order to demonstrate specific mean flow condition optimization, a single mean flow is selected from the wide flow range, $M = 0.9, T_0 = 290K, P_0 = 1.7atm$.

(c) Wire parameters:

Wire diameters are limited in the lower range by structural, operability, and manufacturing considerations, and in the upper limit by the decreased circuit resistance, and reduced frequency response. Moreover, to prevent oxidation of a tungsten wire, the upper temperature is not to exceed $250°C$ [1]. Thence, the ranges of wire properties for the optimization are:

$$\mathbf{5\mu m \leq d_w \leq 10\ \mu m}$$
$$\mathbf{370K \leq\ T_w \leq 510K}$$

RESULTS

The input parameters for the optimization are the mean flow quantities of Mach number, total temperature, and total pressure (corresponding to distinct velocity, density and temperature conditions) Probes comprising of 3 and 4 wires are examined. The number of wire diameters is limited in order to shorten the computation time. The specific diameters considered are $5, 6.5, 8.5, 10[\mu m]$, and the wire temperatures examined are $370{:}\,10{:}\,510[K]$. The overall number of combinations with repetitions for 3 and 4 wire probes are 34,220 and 590,295 respectively.

(a) Wide range of mean flow conditions

For a 4-wire probe, Figure 1 presents the inverse of the condition number with respect to a broad range of flow conditions and probe wire diameter and temperature combinations. Clearly, some probe wire diameter and temperature combinations produce better condition numbers overall; higher inverse condition numbers are more desirable.

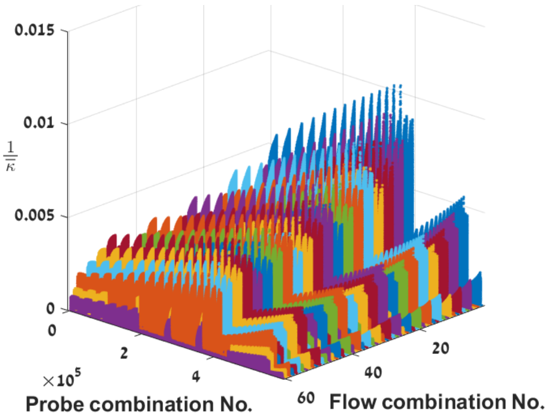


Figure 1 – Inverse condition number of all 4 wire probe combinations for broad range of mean flow properties

As a desired potential objective for optimization, the probe mean condition number and its variance with respect to differing flow conditions are charted in Figure 2. The results are normalized by the minimum mean condition number (κ_{min}) and the minimum variance (σ^2_{min}) of a probe in the entire population. Evidently, the σ^2 and $\kappa(\underline{A})$ parameter are positively correlated (improving one also improves the other). Thus, minimizing one is sufficient for optimization of both variables, corresponding to the desired region in the top right corner of the figure. Therefore, it is sufficient to choose $\kappa(\underline{A})$ as an optimization objective.

In this light, other important parameters for the probe optimization are the condition number $\kappa(\underline{A})$, as well as the $\kappa(\underline{A})\left|\left|\underline{\Delta A}\right|\right|$ term, Eq. (23) and Eq. (24) respectively. Figure 3 charts a map of inverse normalized mean condition number with respect to inverse normalized mean norm of matrix perturbation for all 4-wire probes considered. Evidently, the $\left|\left|\underline{\Delta A}\right|\right|$ and $\kappa(\underline{A})$ parameters are also positively correlated; so minimizing only one of the variables is sufficient. Similarly, it is possible to select the multiplication of the two quantities $\kappa(\underline{A})\left|\left|\underline{\Delta A}\right|\right|$ as an objective for the optimization.

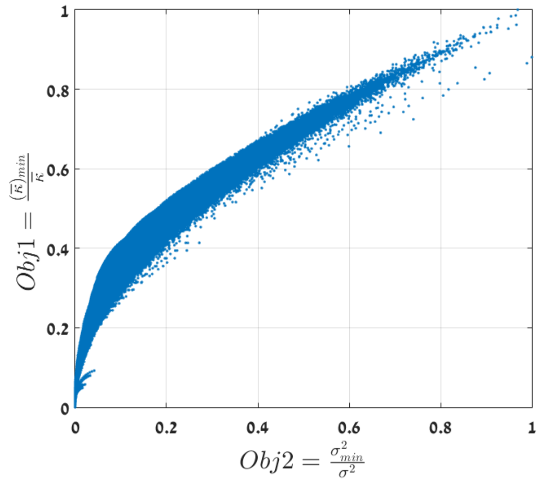


Figure 2 – Map of inverse normalized mean condition number vs. inverse normalized mean norm of matrix perturbation for all 4-wire probe combinations

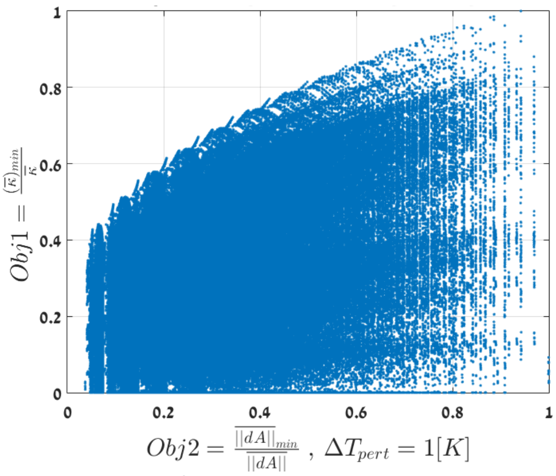


Figure 3 – Map of inverse normalized mean condition number vs. inverse normalized mean norm of matrix perturbation for all 4-wire probe combinations

Considering this corollary along with the significance of matrix norm, the value function described in Eq. (2525) is the natural choice for optimization. The examined weighting combinations of the following results are presented in Table 1.

Table 1: Weighting of Value Functions

Value Function #	*A*	*B*
1	*0.99*	*0.01*
2	*0.70*	*0.30*
3	*0.50*	*0.50*
4	*0.40*	*0.60*
5	*0.10*	*0.90*

Figure 4 portrays a map of the two objectives $\frac{\overline{\left(\kappa(\underline{A})\left\|\underline{\Delta A}\right\|\right)_{min}}}{\overline{\kappa(\underline{A})\left\|\underline{\Delta A}\right\|}}, \frac{\overline{\left\|\underline{A}\right\|}}{\overline{\left\|\underline{A}\right\|}_{max}}$, for all possible 4-wire probe combinations.

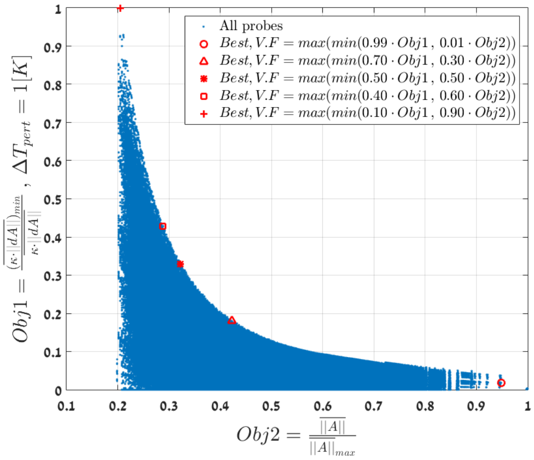


Figure 4 – Map of the two objectives for all 4-wire probes combinations

In this optimization, Obj1 and Obj2 are both maximized. The most desirable probe is in the top right corner of the chart, and hence, there are no universally dominant probes; instead they are scattered along a convex Pareto front. The optimization results for the 5 different value functions are superimposed by red symbols.

Selecting the desired probe from the Pareto front requires analyzing in detail the characteristic performance. Although it is possible to treat each objective as a criteria, the critical threshold to the decoupling is associated with the invertability of the matrix which is characterized by the condition number. Therefore, the condition numbers of the best 4-wire-probes are charted for the various value functions across the considered flow conditions, Figure 5a-e. The particular traits of the resulting probes can be found in Table 2.

Table 2: Best 4-wire Properties for Value Functions

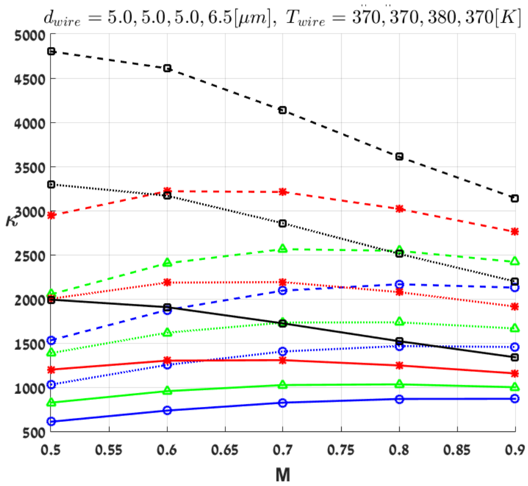
Val	d_{wire} $[\mu m]$				T_{wire} [K]			
1	5	5	5	6.5	370	370	380	370
2	5	5	10	10	410	430	400	450
3	5	5	10	10	430	460	430	460
4	5	5	10	10	450	460	440	480
5	5	5	10	10	500	510	500	510

For value function 1, Figure 5a presents a choice favoring large matrix norm over a minimal $\kappa(\underline{A})\left\|\underline{\Delta A}\right\|$. The ensuing probe consists of 5µm, 5µm, 5µm, and 6.5µm wires at 370K, 370K, 380K, and 370K respectively. Although the voltage sensitivity to changes in flow properties is amplified, the resulting condition numbers are mostly in the 1000-5000 range. This deems the configuration inadmissible.

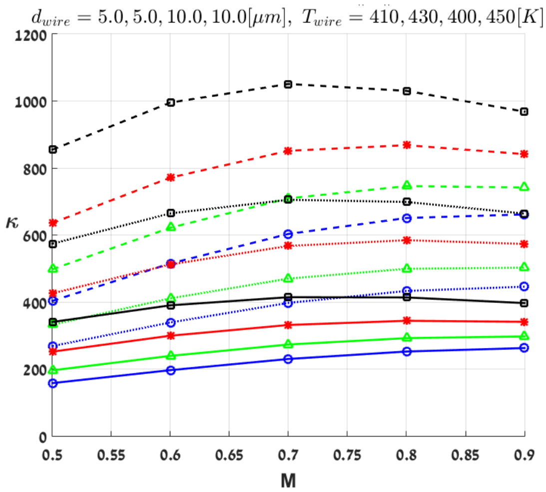
Figure 5b-d are charts presenting the condition number for the best probes resulting from value functions 2-4. These configurations all yield wire diameters as far apart as possible at the limits of the constraint, two 5µm and two 10µm wires. The corresponding wire temperatures are spread over the intermediate range of 400K-480K. The wires with identical diameter do not correspond to the same temperature. The system of equations does not seem to favor redundant wire systems. Throughout the entire flow range, the corresponding condition numbers are less than 1000 for all three probes. Expectedly, due to weighting that favors the lower condition-number probe preferences, the value function 4 yields slightly lower $\kappa(\underline{A})$. In general, the condition numbers rise for higher Mach number and higher total temperature flows. Moreover, there exists a strong and adverse effect of density: the higher the density, the bigger the condition number. However, all three probes are acceptable from an invertability perspective.

For value-function 5 that strongly favors $\kappa(\underline{A})$ over the norm, the results are presented in Figure 5e. The probe configuration associated still yields two 5µm and two 10µm wires, with temperatures of 500K and 510K pairs. Considering that the optimization has reached the upper limit in the wire temperature constrain, and that the temperature values are very close to each other, the probe configuration is less desirable.

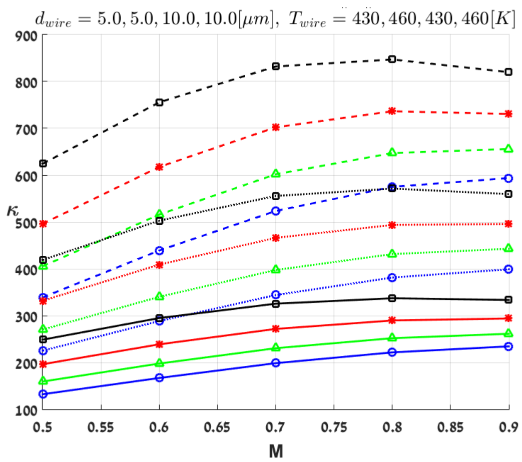
In summary, according to the admissible probes from value function 2-4, spreading the choice of wire diameters to pairs in the upper and lower constraints is recommended. Moreover, the temperatures have a tendency to be sufficiently far apart from one another (10K-40K) concentrated around ~450K.



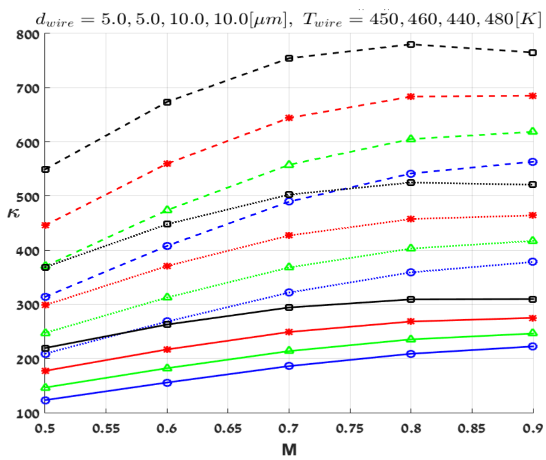
a) Best Probe for $Val1, A = 0.99 ; B = 0.01$



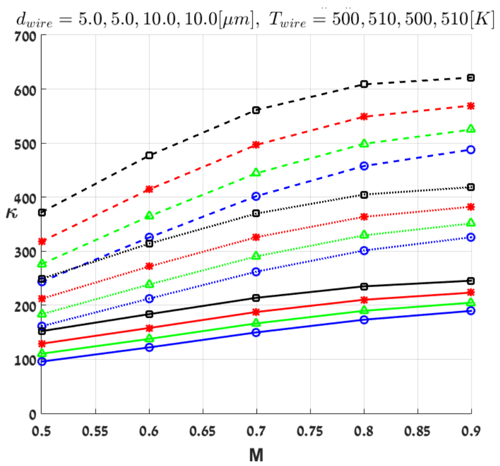
b) Best Probe for $Val2, A = 0.7 ; B = 0.3$



c) Best Probe for $Val3, A = 0.5 ; B = 0.5$



d) Best Probe for $Val4, A = 0.4 ; B = 0.6$



e) Best Probe for $Val5, A = 0.1 ; B = 0.9$

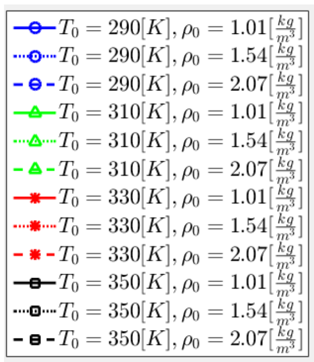


Figure 5 - Condition numbers of best 4-wire probe for all value functions

In order to assess the dependency of the optimization on the number of wires in the probe, Figure 6 is a representative example, contrasting the condition numbers of best 3-wire and 4-wire probes for value function 4 at $T_0 = 350K$. For value function 4, the best 3-wire probe wire diameters and temperature are $d_w = 5{,}5{,}10[\mu m]\,; T_w = 440{,}480{,}450[K]$ respectively. In general, the condition number for a 3 wire probe is slightly worse than a 4-wire probe. In order to demonstrate the ramifications associated with noise amplification, Figure 7 directly charts the $\kappa(\underline{A})\frac{||\Delta A||}{||\underline{A}||}$ term for the same probes. It is apparent that the noise amplification is lower for conditions producing lower condition number. And, in general, 4 wire probes result in reduced noise amplification. Similar trends can be observed for other value functions.

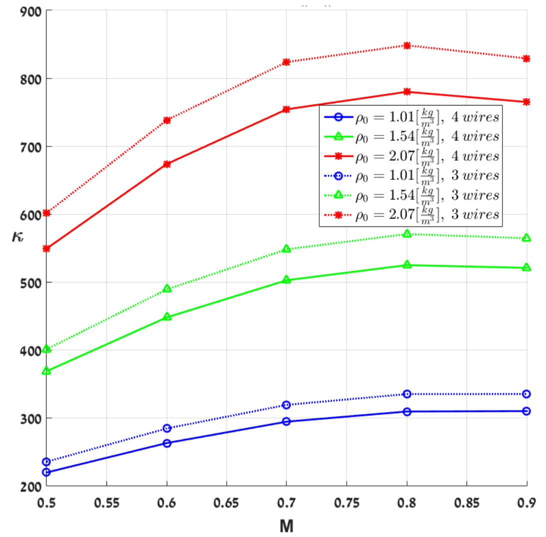


Figure 6 – Contrasting the Condition numbers of best 4-wire and 3-wire probes for Value function 4 at $T_0 = 350K$

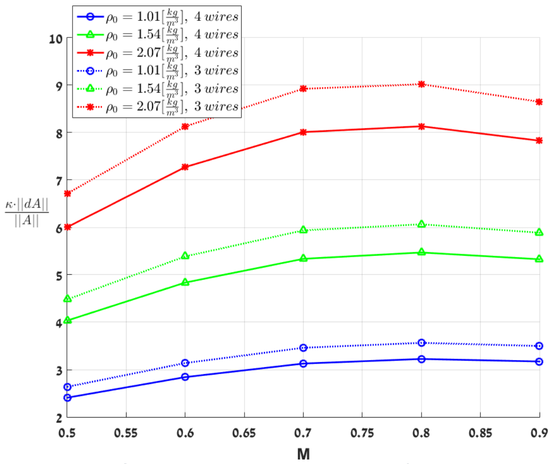


Figure 7 – Contrasting the error amplification due to sensitivity matrix perturbation for best 4-wire and 3-wire probes, Value function 4 at $T_0 = 350K$

(b) Specific mean flow condition

The particular flow condition selected for the optimization $(M = 0.9, T_0 = 290K, P_0 = 1.7atm)$ yields a relatively high condition number. For example, in the wide flow range optimization, the best probe of value function 4 is: $d_w = 5, 5, 10, 10[\mu m]\,; T_w = 450, 460, 440, 480\ [K]$ respectively. The corresponding condition number is 563, and $\kappa(\underline{A})\frac{||\Delta A||}{||\underline{A}||} = 3.76$. A single-flow optimization for the same value function, resulted in a probe with same wire diameters, but different wire temperatures. $d_w = 5{,}5{,}10{,}10[\mu m]\,; T_w = 460{,}490{,}450{,}510[K]$ respectively. Its condition number is 529, and $\kappa(\underline{A})\frac{||\Delta A||}{||\underline{A}||} = 3.24$. Slightly improving the flow-specific condition number and the over-all error criteria.

SUMMARY AND CONCLUSIONS

In order to select CTA probe's wire properties such that the resultant system will have high sensitivity to flow properties and low noise amplification, an optimization methodology is presented. Based on empirical $\mathrm{Nu} - Re - M$ and $\eta - Re - M$ correlations, the technique is applicable to the compressible M regime with total pressures and temperatures characteristic to a highly loaded fan stage. Towards future implementation in various turbomachinery and propulsion applications, some general design guidelines can be formulated:

- The condition number $\kappa(\underline{A})$, which characterizes the invertability and the well-posedness of the sensitivity matrix, is the dominant characteristic quantity in this problem. Typically, systems with $\kappa(\underline{A}) > 1000$ are tending towards being ill-posed and therefore are not advisable.
- Selection of the value function can be simplified by the notion that the change in variance $(\sigma^2)$and $||\underline{\Delta A}||$ for different probe combinations is positively correlated with the corresponding condition number, $\kappa(\underline{A})$. Thus, the need for more independent objectives is obviated.
- Compromising between the two objectives associated with minimum error amplification and maximizing the sensitivity matrix, the chosen value function in max-min formulation is presented in Eq. (25).
- Based on the trends observed in the optimization, the probe should consist of wire diameter combinations at the extremes of the available range (in this case $5\ \mu m$ and $10 \mu m$). Therefore, the suggested 4-wire probe consist of 5-5-10-10 μm wires. This selection allows optimizing the probe to various flow conditions by solely adjusting the wire temperatures.

- The corresponding wire temperatures are spread over the intermediate range of 400K-480K. The wires with identical diameter do not correspond to the same temperature. Moreover, the temperatures should be sufficiently far apart from one another (10K-40K) concentrated around ~450K.
- As expected, 4-wire probes present less noise amplification than 3-wire probes, due to the overdetermined nature of the system.
- The presented methodology can be utilized either for globally optimal probes or for particular flow ranges.

FUTURE WORK

The current optimization delivers a population of probes located on the Pareto line, all of which maximize the proposed objectives. Still, there is no "single best probe" selection mechanism. Thus far, it is up to the user to select the weights, which are not easily determined as the comparison between different objectives is often not "apples to apples". Therefore, further search for constrains is needed. One prospective option is a synthetic wire voltage simulation with artificial noise. This may enable the optimization of α and ζ. Lastly, in order to establish the validity of the proposed methodology, experiments will be conducted to compare the flow decoupling using optimal and non-optimal probe combinations.

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