ON THE CAPABILITY OF FAST RESPONSE TOTAL PRESSURE PROBES TO MEASURE TURBULENCE KINETIC ENERGY

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ABSTRACT
This paper presents a study on the performance of fast-response total pressure probes to turbulent flows. Theoretically, unsteady pressure measurements could provide relevant information on the turbulent field; in practice, the unsteady aerodynamic response of the probes dramatically affects the turbulent structures and produces a loading effect of difficult evaluation and subsequent correction.

In a previous paper of the same authors the qualitative relevance of fast-response probes to measure turbulence was demonstrated in comparison with LDV measurement in turbomachinery flows. The present paper, instead, wants to assess the quantitative accuracy of turbulent data derived by pressure probes, through a critical comparison of the time-resolved total pressure measurements and hot wires in an open jet. The desired turbulence level and the isotropy of the jet is achieved thanks to the use of a properly designed turbulence generator system.

The jet has been completely characterized through the use of a hot wire technique, that allows to reconstruct the parts of the Reynolds stress tensor by applying a normal wire probe at different times. The turbulence evolution along the jet is studied to investigate the turbulence decay; in particular, special attention is paid to the isotropy of the turbulent structures. Total pressure probes, indeed, do not allow to isolate the contributions of the single components, and thus, reliable quantitative measurements on the turbulent kinetic energy can be achieved only for isotropic turbulence. Alongside integral estimates on the turbulence level and spectral analyses on the single hot wire measurements have also been performed, to investigate the structure of turbulence in the core of the jet.

Once the jet has been completely characterized, unsteady total pressure measurements have been performed in the same position of the hot wires, and a direct comparison is developed. At first, a refined method is proposed to compute the turbulence level, as well as the instantaneous turbulent fluctuation of velocity. Integral evaluation as well as frequency-domain analyses have been performed, to allow a direct comparison.

The results indicate that an isotropic core actually exists in the initial part of the jet: the streamwise and the binormal turbulent velocity components are quantitatively very close, and the turbulence decay rate is in close agreement to that observed in isotropic region of confined jets. In these regions, the total pressure probe is found to provide accurate results, but only if assumed that all the three components of the turbulent velocity are isentropically recovered on the probe head. In other words, in isotropic turbulence the total pressure probe seems to be sensitive to the overall turbulent kinetic energy rather than to only the streamwise component.

INTRODUCTION
Fast-Response Aerodynamic Pressure Probes (FRAPP), based on the matching between the technology of pneumatic multi-hole aerodynamic probes and that of piezoresistive pressure sensors, represent a mature and a relatively complete measurement technique for turbomachinery applications, and it is now ready for large-scale industrial use. FRAPP allow, indeed, to detect the deterministic component of unsteadiness that typically affects turbomachinery flows, and thus it offers dramatic simplifications in measuring the unsteady pressure/velocity field downstream of turbomachinery rotors. This is of particular relevance when rotating frame instrumentation is not available or, as in high-speed application, is not feasible. Classical references on unsteady pressure measurements with probes are Kupferschmied et al., 2000, Ainsworth et al., 2002, and Brouckaert, 2004.

The development of FRAPP has been directed towards extreme miniaturization: external, to reduce the probe blockage and improve the spatial resolution, and internal, to minimize the line-cavity system and thus improving the dynamic response. Such an effort has led to extremely small (~1 mm) and fast (~100 kHz) probes; these performances indicate that FRAPP could be – in principle – extended to the measurement, alongside the deterministic unsteadiness, of some turbulence properties of the flow.

In fact, since the very first trials of this measurement technique (Heneka, 1983), the FRAPP were tentatively applied also to turbulence measurements, by accounting for the non-deterministic components of the pressure readings of the probe. This was possible since these probes, encapsulating three or four sensors, capture the effective time-resolved pressure distribution on the probe head, and from this the actual instantaneous flow
field can be estimated. Unfortunately the agreement between turbulence measurements performed with FRAPP and with hot wires or laser-doppler velocimetry was found to be poor (Ruck, 1988), probably due to the relatively large probe head dimension (about 5 mm).

After these first attempts, some research groups chose to minimize the probe dimension by drastically reducing the number of sensors, thanks to the virtual multi–sensor probe concept. According to this philosophy, three pressure taps/sensors can be replaced by three rotations of a single tap/sensor around the probe axis; thus two-dimensional measurements can be attained with a single-sensor probe (Kupferschmied et al., 2000), while three-dimensional measurements can be obtained with two single-sensor probes (Pfau et al., 2002) or with a two-sensor probe (Porreca et al., 2007, Persico et al., 2010). Unfortunately virtual operation modes are based on the combination of measurement performed at different times; therefore, the periodic component of the unsteadiness (phase-resolved flow) can be captured, but the actual instantaneous flow is not measured. As a consequence, these kind of probes should not be able to detect any non-periodic component, i.e. transients and turbulent phenomena.

In fact, the core of the problem is in the angular sensitivity of the probe head; if the probe has a large insensitivity to the flow angle, and the probe tap is aligned with the time/phase-averaged velocity vector, the actual instantaneous total pressure is measured by the probe. By exploiting this concept, Wallace and Davies, 1996, proposed a method to measure the streamwise turbulent component from a total pressure probe. More recently, Porreca et al., 2007, have shown that, for incompressible flows, turbulence measurements can also be performed with a two-sensor probe, but without any comparison with other measurement techniques. Persico et al., 2008, applied these concepts to a single-sensor cylindrical probe and, assuming isotropic turbulence, showed a good agreement with LDV measurements performed downstream of a transonic turbine stage. The present paper, in the wake of the previous one, is focused on a more rigorous quantitative analysis of the capabilities of a fast response total pressure probe, characterized by a very large insensitivity range, in measuring the turbulence kinetic energy of a turbulent jet in comparison with hot wire anemometry.

**INSTRUMENTATION**

**Hot Wire Anemometry**

The extreme miniaturization (wire diameter ~5 μm) and the very high dynamic response (~30 kHz) make the constant temperature anemometry (CTA) the most suitable tool for turbulence measurements, if applied in single-sensor/wire probes. DISA normal and slanted single-sensor probes were applied in this research, with the aim to fully characterize the turbulent structures of the free jet and thus to provide the reference for the total pressure probe data. However, only the results obtained with the normal wire probe will be discussed in the following.

**Fast-response total pressure probe**

A fast-response miniaturized total pressure probe (FRTPP) was developed on the basis of a commercially available piezoresistive transducer (Kulite, model XCQ-062, FS = 25 Psi, temperature compensated, extended uncertainty ±80 Pa). The transducer is encapsulated in a metallic tube with a diameter of 3 mm. To enhance the flow angle insensitivity (Dénos, 2002), the transducer’s membrane is mounted with a recess of 0.8 mm. In Figure 1 the pressure coefficient $K_{Pt} = \frac{Pt - P}{Pt - Ps}$ is plotted against the yaw angle; within the yaw angle range ±22°, total pressure measurements variations remain largely inside the measurement uncertainty, while for the flush mounted sensor the insensitivity to the yaw angle reduces to less than ±10°. Once it is aligned with the time-averaged flow direction, the probe is actually a time-resolved measurement device for total pressure measurements if the flow angle fluctuation remains within ±22°.

This improved aerodynamic performance is obtained at the expense of a reduction of frequency response with respect to the flush-mounted configuration (order of magnitude hundreds of kHz). Dynamic calibrations of the FRTPP in the shock tube, whose results are reported in Figure 2, indicate a good linearity in the frequency response, with a dynamic behaviour typical of an under-damped second order linear system, characterized by a natural frequency of 49.5 kHz and a damping factor of about 0.1.
TURBULENCE ESTIMATION FROM UNSTEADY TOTAL PRESSURE MEASUREMENTS

The time-resolved nature of the measurements performed with the total pressure probes allows to directly derive the unresolved total pressure unsteadiness from the instantaneous data. Once the deterministic component is extracted, the unresolved component is computed by subtracting the deterministic component to the instantaneous signal. The full procedure, valid for both total pressure probe and cylindrical aerodynamic probes, is reported extensively in Persico et al., 2008.

For the aims of the present research, the unresolved total pressure must be somehow converted into a turbulent quantity. The way the probe converts the turbulent velocity fluctuations into total pressure random unsteadiness is the key of the present research, and it is now discussed in detail. Assuming incompressible one-dimensional flows, the total pressure can be expressed as (with clear meaning of the symbols):

\[ p_T = p_s + \frac{1}{2} \rho V^2 \]

By splitting the components into a time/phase mean and fluctuating (turbulent) component, we get:

\[ \overline{p_T} + p_T' = \overline{p_s} + p_s' + \frac{1}{2} \rho (\overline{V} + V')^2 = \overline{p_s} + p_s' + \frac{1}{2} \rho V^2 + \frac{1}{2} \rho V'^2 + \rho \overline{V} V' \]

from which we can derive an expression for the fluctuating total pressure:

\[ p_T' = p_s' + \frac{1}{2} \rho V'^2 + \rho \overline{V} V' \]

Introducing the Hinze’s expression for the turbulent fluctuation (Hinze, 1975), valid for isotropic turbulence, we can obtain a formulation only dependent on the fluctuating velocity, that allows to directly convert the total pressure random unsteadiness in turbulent velocity:

\[ V'^2 + \frac{\overline{V}}{1,2} V' - \frac{p_T'}{1,2 \rho} = 0 \quad (1) \]

Once the instantaneous turbulent velocity is known, a spectral analysis can be performed on the signal to highlight the structure of turbulence as measured by the total pressure probe, to be compared with the one derived from the hot wires. In particular, the power spectrum density (PSD) is used to represent the signal in the frequency domain. Calling \( f \) the frequency, \( df \) the frequency resolution, \( A(f) \) the Fourier transform of the signal, and \( A^*(f) \) the complex conjugate of \( A(f) \), the PSD is defined as follows:

\[ PSD = \frac{A(f) A^*(f)}{df} \]

By making the square of the equation (1), taking the mean and neglecting the mean of the cubic terms...
HOT WIRE SIGNAL ANALYSIS

The basic concept of hot wire anemometry is to relate the velocity of the incoming flow to the amount of cooling induced on the wire. For CTA this means to link the so-called cooling velocity to the voltage required to keep constant the operating temperature of the wire. The most popular formulation of this link is the King’s law:

\[ E^2 = E_0^2 + BQ^n \]

with \( E \) the voltage, \( E_0 \) the voltage at zero velocity, \( Q \) the cooling velocity, \( B \) and \( n \) two calibration coefficients. A typical calibration curve is reported in Figure 3.

The cooling velocity appearing in the King’s law differs from the actual velocity due to the angular sensitivity of the probe to the three velocity component in a three-dimensional flow (turbulence is, by definition, three-dimensional). The most comprehensive expression is provided by the Jorgensen’s law. With reference to the scheme in Figure 4, in which an intrinsic reference system is defined with respect to the wire, the cooling velocity can be expressed as:

\[ Q^2 = U_n^2 + k^2 U_t^2 + h^2 U_b^2 \]

with \( U \) the flow velocity, \( k \) and \( h \) two angular calibration coefficients that must be estimated through angular calibration. Full details on the calibration procedure and data reduction techniques can be found in Perdichizzi et al., 1990. In the following, only a brief review of the procedure is reported, applied to normal wire probes.

By applying the usual decomposition to the King’s law, an expression for the fluctuating cooling velocity can be derived, directly linked to the fluctuating voltage and depending on the mean components.

\[ \overline{q^{1/2}} = \left[ \frac{2c}{nBq^{n-1}} \right]^{1/2} \]

Once this quantity is computed, the Jorgensen’s law is used to derive the actual velocity components. The multiple unknowns deriving from the application of a multidimensional expression to a single-wire probe requires multiple rotation of the probe around its axis. However, the virtual nature of this procedure does not affect the time-resolved character of the hot wire measurements, as it will be clear in the following.

At first an intrinsic reference frame for the flow is introduced according to the scheme reported in Figure 5 (note that for normal wire the \( \alpha \) angle is 0 deg); the rotation angle of the probe around its axis is also reported. In this scheme, the mean flow is intended to be directed along \( X_2 \) and the probe is rotated around the \( X_1 \) axis. Expressions can now be obtained for the three intrinsic components of the wire, depending on the rotation angle:
\[ U_n = U_1 \; \quad U_t = U_2 \cos \varphi - U_3 \sin \varphi \; \quad U_b = U_2 \sin \varphi - U_3 \cos \varphi \]

Implementing these contributions in the Jorgensen’s Law, the cooling velocity can be expressed in terms of the velocity components and of the rotation angle of the probe:

\[ Q = U_1^2 + (k^2 \cos^2 \varphi + h^2 \sin^2 \varphi) U_2^2 + (k^2 \sin^2 \varphi + h^2 \cos^2 \varphi) U_3^2 - (k^2 - h^2) \sin 2\varphi \ U_2 U_3 \]

By introducing the usual decomposition in mean and fluctuating component, an expression is finally achieved to link the root mean square of fluctuating cooling velocity (computed from the voltage) and that of the turbulent velocity components:

\[ \overline{q'^2} = (k^2 \cos^2 \varphi + h^2 \sin^2 \varphi) \overline{u_2'^2} + \frac{1}{4} \frac{(k^2 - h^2)^2 \sin 2\varphi}{k^2 \cos^2 \varphi + h^2 \sin^2 \varphi} \overline{u_3'^2} - (k^2 - h^2) \sin 2\varphi \overline{u_2 u_3'} \]

Once the calibration coefficients \( h \) and \( k \) are known for the different orientation angles of the probe, three terms of the Reynolds stress tensor can be estimated by means of three rotations of the probe. The trends obtained – after calibration – for the coefficients appearing in the equation reported above are represented in Figure 6. All the coefficients have relatively high values at least in some regions, and this guarantees high sensitivity. In practice, the three rotations were chosen at 45°, 90° and 135°. The degree of anisotropy of the turbulent field will therefore evaluated by comparing the values of the quadratic terms, and assuming that the third quadratic component (along \( X_1 \) axis) follows the behavior of the previous ones.

Alongside the estimation of the Reynolds stresses, the instantaneous hot wire signals were also used to derive information on the turbulent scales and turbulent structures in frequency domain. To this aim, the probe was mounted aligned co-axial with the jet (\( n \) coincides with \( X_2 \)), in such a way that only the normal turbulent component contributes to the fluctuating cooling velocity. From these data the PSD of the turbulence intensity in streamwise direction can be evaluated, in the same way of what done for the total pressure random data.

**NOZZLE DESIGN AND JET CALIBRATION**

The comparison between the two techniques discussed above was developed on an open jet. Being developed for this application, special attention was paid in the design of the nozzle and of the turbulence generator and a full calibration of the jet was performed.

A plane nozzle was designed, using circular arcs as lateral surfaces. At the end of the contraction section, a turbulent generator was placed. A number of double-array, circular-bar, grid-shaped turbulent generators were designed and tested, following the indications of Roach, 1987, to minimize the degree of anisotropy of the turbulence. To further improve the isotropy of the jet, a further contraction was realized downstream of the grid according to the method proposed by Comte-Bellot and Corrsin, 1966.

At first, the time-averaged characteristics of the jet were investigated by traversing, in both axial and
transversal direction, a pneumatic total pressure probe. To remain inside the incompressible flow regime, the Mach number of the jet was limited to 0.25.

The results of this analysis are summarized in Figure 7, in which the velocity profile measured with and without the grid are reported for different distances from the nozzle discharge section and for a Mach number of the jet equal to 0.2. The measurements indicate that a large region of mixing exists in the lateral part of the jet, whose development is much stronger – and faster – than that would be achieved in confined jets. Measurements show that the grid has an effect also on the mean velocity, producing a more flat profile across the jet. The lower velocity value is due to the losses across the grid.

Hot wires were then applied to characterize the turbulence of the jet. Only the turbulence properties in the core of the jet were investigated, so the hot wire probe was placed in the center of the jet and only traversed along the streamwise direction. At first the probe was aligned with the jet, to derive the instantaneous velocity signal, and then it was placed normal to the jet in order to derive the Reynolds stress, and to evaluate the degree of anisotropy.

In confined jets, once a certain turbulence intensity is generated through a grid, three regions can be observed. Just downstream of the grid, the wakes of the bars are still visible and the turbulence is highly anisotropic. In the intermediate region the wakes are diffused, local isotropy is nearly achieved and the turbulence decays according to mainly inertial processes, following the so-called Power Law Decay, i.e.:

$$\frac{\overline{u'^2}}{V^2} = A \left( \frac{x}{M} \right)^{-n},$$

$M$ being the transversal dimension of the jet.

Very far from the grid, the viscosity of fluid acquires importance and the degree of anisotropy enhances again. In the case of open jets, this latter component is relevant, due to the mixing process that occurs between the jet and the outer environment. Therefore, a detailed analysis of the axial evolution of the jet was performed to indentify the extension of the isotropic region.

To identify these different zones in the present jet, turbulent component of the velocity in streamwise direction $I_u$ is considered, defined as:

$$I_u = \frac{V'}{V} = \sqrt{\frac{\overline{u'^2}}{V}}$$

In Figure 8 the evolution of the streamwise turbulence intensity, measured in the core of the jet, with the normalized distance from the nozzle is reported for four operating Mach numbers (all of them in nearly
incompressible flow regime). On the basis of the measured trends, two zones can be recognized: for $x/M < 25$, an almost linear decay (on a double logarithmic scale) is observed; for $x/M \geq 30$, the intensity tends to increase significantly, with a growing rate as the distance increases.

In the first of these two zones one can recognize the “inertial” region, where the Power Law Decay represents fairly well the dynamics of turbulence, while in the second one it is possible to recognize the “viscous” region, where the mixing with the external environment contributes to produce turbulence. These results indicate that, if any, a nearly isotropic region could be found only for $x/M \leq 25$.

To evaluate the degree of anisotropy of the turbulent field, an analysis of the Reynolds stress components was performed, with the probe mounted normal to the jet, as described in the previous section. In Table 1 the results of the Reynolds stress analysis are reported for a Mach number of the jet equal to 0.25, with $u_n$ and $w_n$ the streamwise and binormal components respectively. Alongside the measured values, the repeatability of the experimental data is also reported in Table 1. Results indicate that the degree of anisotropy, measured by the ratio between the stresses, is very limited in the first part of the jet, where the Power Law Decay is valid, while, for $x/M \geq 30$, a relevant difference arises between the components.

This analysis allows to conclude that the jet is fairly well isotropic for $x/M \leq 25$. In this region, therefore, most of the comparison with the fast response total pressure probe will be developed.

RESULTS AND DISCUSSION

The fast-response total pressure probe was applied downstream of the nozzle in the same positions of the hot wire probe. The pressure signal was sampled at 100 kHz, and the data were digitally compensated and then filtered at 30 kHz to guarantee coherence with the frequency response of the hot wires. Then, equations (1) and (2) were used to derive both the turbulence intensity $I$ and the PSD from total pressure data.

At first the evolution of turbulence intensity along the jet was considered, and the main results are reported in Figure 9 against hot wire data in the same conditions ($M = 0.25$). From the qualitative point of view, the total pressure measurements indicate a very similar trend, with a linear decay in the first part, and a steep rise in the second part. However, from the quantitative point of view the pressure measurements seem to overestimate significantly the turbulence intensity measured by the hot wire. Rather surprisingly, the maximum discrepancy is observed just in the isotropic region, where the methodology and the pressure probe itself are expected to behave in a better way.

These results, consistent and repeatable after a long test campaign, suggest that there is something missing in the modeling of the recovery mechanism that occurs on the pressure probe. The probe being characterized by a so large flow angle insensitivity, and the turbulent flow angle fluctuation being small, it should be expected that not only the streamwise turbulent component is recovered, but all the three components are recovered on the probe head. If this is the actual mechanism occurring on the probe head, the one-dimensional method proposed above must be corrected to take into account the role of all the three velocity components. In other words, the fluctuating velocity – and thus the turbulence intensity – to that the total pressure probe is sensitive – is given by the combination of all the three turbulent components, i.e.:

$$ V' = \sqrt{\frac{\overline{u_1'^2}}{I_{PP}}} $$

In general, no clear link seems to exist between the measurement performed with the total pressure probe and that of the hot wire. However, if the turbulence can be considered isotropic, the expression above can be highly simplified, and written only in terms of the streamwise component:

$$ \frac{I_{PP}}{V} = \sqrt{\frac{\overline{u_1'^2} + \overline{u_2'^2} + \overline{u_3'^2}}{V}} $$

Table 1: Reynolds stresses and degree of anisotropy in the core of the jet

<table>
<thead>
<tr>
<th>$x/M$</th>
<th>$\sqrt{\overline{u_1'^2}}$</th>
<th>$\sqrt{\overline{u_2'^2}}$</th>
<th>$\sqrt{\overline{w_n'^2}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>1.10±0.01</td>
<td>1.08±0.01</td>
<td>1.02±0.01</td>
</tr>
<tr>
<td>25</td>
<td>1.01±0.01</td>
<td>0.96±0.01</td>
<td>1.06±0.02</td>
</tr>
<tr>
<td>30</td>
<td>1.03±0.01</td>
<td>0.94±0.01</td>
<td>1.10±0.01</td>
</tr>
<tr>
<td>35</td>
<td>1.16±0.02</td>
<td>0.97±0.03</td>
<td>1.21±0.06</td>
</tr>
<tr>
<td>40</td>
<td>1.32±0.02</td>
<td>1.15±0.02</td>
<td>1.15±0.03</td>
</tr>
<tr>
<td>45</td>
<td>1.53±0.03</td>
<td>1.31±0.04</td>
<td>1.17±0.05</td>
</tr>
</tbody>
</table>
This consideration allows to re-establish a connection between the measurements performed with the hot wire and the total pressure probe, at least in the isotropic region. If applied to the present data, a relevant quantitative agreement is achieved between the measurement technique for \( x/M \leq 25 \). For larger distances from the nozzle, where the degree of anisotropy becomes relevant, the quantitative accuracy rapidly decays.

A further support of the present conclusion is achieved when comparing the PSD of the hot wire and of the total pressure signals in the isotropic region, reported in Figure 11. The PSD of the hot wire signal shows the typical trend of the turbulence energy cascade, with a large inertial band. When the total pressure signal is corrected to consider the recovery effect, a relevant quantitative accuracy is achieved between the two spectra. This means that not only the overall turbulence intensity, but also the general structure of the turbulence scales is estimated accurately by the total pressure probe.

**CONCLUSIONS**

This paper presents a study on the performance of fast-response pressure probes to turbulent flows has been presented. In particular, the quantitative accuracy of turbulent data derived by pressure probes has been assessed through a critical comparison of the time-resolved total pressure data and hot wire data in an open jet. The jet, designed to achieve the maximum level of isotropy in the turbulence field, was completely characterized.
through the use of a single-sensor hot wires, applied with virtual operation more to reconstruct some components of the Reynolds stress tensor. The turbulence evolution along the jet is studied, to investigate the turbulence decay, the degree of anisotropy and the structure of the turbulent field through frequency domain analysis. The results indicate that an isotropic core actually exists in the initial part of the jet: the streamwise and the normal turbulent velocity components of the are quantitatively very close, and the turbulence decay rate is in close agreement to that observed in confined jets.

Once the jet has been completely characterized, unsteady total pressure measurements have been performed in the same position of the hot wires, and a direct comparison is developed. At first, a refined method is proposed to compute the turbulence level, as well as the instantaneous turbulent fluctuation of velocity. Integral evaluation as well as frequency-domain analyses have been performed, to allow a direct comparison. In the region of isotropic turbulence, the total pressure probe is found to provide accurate results, but only is assumed that all the three turbulent components are completely recovered on the probe head. In other words, the total pressure probe seems to be sensitive to directly the overall turbulent kinetic energy rather than to only the streamwise component.

REFERENCES


