ABSTRACT

This paper presents a calibration and data reduction technique for three-hole pressure (THP) probes, providing an angular range that is significantly extended. The maximum range varies with the probe characteristics (head geometry, construction angle,…). For instance, in the case of a cylindrical probe with a 30 deg construction angle, the methodology advanced here provides an angular range of ±75 deg. Moreover, using 35 deg cylindrical probes, it is possible to obtain maximum angular ranges close to ±90 deg.

INTRODUCTION

THP probes operating in a “non-nulling” ([1]) mode are usually employed to characterize two-dimensional flows in cascades and turbomachines. With these probes, pressure and velocity magnitudes can be obtained for incompressible flow. They can be designed according to different head geometries (cobra type, trapezoidal,…), although cylindrical shapes are the most typical. In addition, THP probes can be either slow-response (pneumatic) or fast-response (dynamic) probes. For the latter, introducing miniature pressure sensors with high frequency response, it is possible to measure unsteady flows, wakes downstream of moving elements, and even turbulence ([2]). Figure 1 shows a sketch of a fast-response cylindrical THP probe.
with Kulite transducers, has been employed to measure a highly-unsteady flow downstream of a low-speed, single-staged, axial turbomachine.

RESULTS AND DISCUSSION

The top plot in figure 2 represents, as a function of the flow angle $\alpha$, the theoretical pressure coefficients $C_p$ in the holes of a 30 deg, cylindrical THP probe. This figure is symmetrical with respect to the zero-incidence flow angle ($\alpha=0$ deg). The pressure coefficient for the central hole has been taken from the experimental data in Weidman ([4]), corresponding to the pressure distribution around a cylinder for a Reynolds number of $2.3 \cdot 10^4$. We consider that the pressure coefficients are theoretic because the distributions in the lateral holes have been obtained shifting the central distribution $\pm 30$ deg respectively. At first sight, from the distributions shown in the figure, it would be expected to obtain a maximum angular range of $\pm 100$ deg for this probe. At $\pm 100$ deg, whatever angular calibration coefficient is defined, double solutions (double points) arise for $C_\alpha$, thus avoiding a univocal determination of the flow angle.

![Fig. 2: Pressure coefficients and traditional angular coefficient for a 30 deg construction angle cylindrical THP probe.](image)

However, in order to avoid the appearance of singular points limiting the angular range of THP probes, a zone-based method should be applied. Hence, it is possible to discriminate different zones along the angular range of the probe, where different angular coefficients $C_\alpha$ are defined for each zone. The major inconvenience is that the flow angle cannot be used to distinguish each zone, because it is unknown a priori. Therefore, relations between the pressure values measured in the holes must be employed instead. With this election, it is possible to distinguish a maximum number of six zones: zone 1 when $P_1>P_2>P_3$, zone 2 when $P_1>P_3>P_2$, zone 3 when $P_2>P_1>P_3$ and so on... Notice that this implies a maximum angular range of $\pm 85$ deg in figure 2. Beyond those limits, the zones are duplicated so they cannot be discriminated. Therefore, the maximum angular range to be attained is established at $\pm 85$ deg (instead of $\pm 100$ deg), and limited due to the existence of duplicated zones, rather than the presence of double points. Depending on the probe characteristics (head geometry, construction angle) the maximum range will be limited by duplicated zones or double points.

On the other hand, to attain the maximum angular range, it is necessary that no singular points arise in the angular coefficient within that range. Otherwise, indeterminations in the calculation of the flow angle will be found. To illustrate this point, the angular coefficient for the traditional calibration has been included in the bottom plot of figure 2. It can be observed that such coefficient has two singular points around $\pm 37$ deg, thus reducing the practical angular range of the probe from the maximum span of $\pm 85$ deg to just a limited range of $\pm 35$ deg.
To confirm the previous analysis, two different zones within the flow angles range have been considered below: zone 1 when \( P_2 > P_3 \) and zone 2 when \( P_3 > P_2 \). For each zone, a different angular coefficient is introduced:

\[
C_\alpha = \begin{cases} 
\frac{P_2 - P_3}{P_1 + P_2 - 2P_3} & \text{when } P_2 > P_3 \\
\frac{P_2 - P_3}{P_1 + P_3 - 2P_2} & \text{when } P_3 > P_2
\end{cases}
\]  

(1)

At first glance, it is possible to define an angular coefficient based on any pressure combination if such combination assures that \( C_\alpha \) is independent of both static and dynamic pressures. As a consequence, the angular coefficient is exclusively a function of the flow angle. Figure 3 shows the angular coefficient defined in (1), compared to its traditional formulation. With the zone-based method, there are no singular points arising until ±80 deg, allowing to attain an operative angular range of ±75 deg. In the case of the traditional calibration, only a limited span of ±35 deg is fulfilled.

![Fig. 3: Traditional and zone-based angular coefficients for a 30 deg cylindrical THP probe.](image)

It must be remarked that, for attainable angular ranges of ±75 deg, at least one of the three holes of the probe has to be measuring in the separated flow region. In the case of pneumatic probes, this is not a major problem; but it can derive in serious limitations for dynamic probes. Therefore, if measurements under separated flow conditions want to be avoided, the angular range of the probe must be further reduced. In the case of a 30 deg cylindrical THP probe (see figure 3) this means that the operative angular range is limited to ±50 deg (incipient detached flow for the lateral holes). Even so, the range obtained with the new proposal provides a considerable increment with respect to the traditional one.

The transmission of the uncertainty of the sensed pressures towards the flow angle is calculated for the relation (1), following the method proposed by Kline ([5]). Since the flow angle is determined from the pressure values \( P_1 \), \( P_2 \) and \( P_3 \), the uncertainty of the flow angle is estimated from the uncertainty of the pressure measurements according to:

\[
I_{\alpha}^2 = \left( \frac{\partial \alpha}{\partial P_1} \right)^2 I_{P_1}^2 + \left( \frac{\partial \alpha}{\partial P_2} \right)^2 I_{P_2}^2 + \left( \frac{\partial \alpha}{\partial P_3} \right)^2 I_{P_3}^2
\]  

(2)
In (1), it was established that $C_\alpha$ is only a function of the flow angle that must be determined from the calibration of the probe. Unfortunately, the flow angle is thus expressed implicitly in a discrete form, so it cannot be resolved directly through an explicit analytical solution. As a consequence, the partial derivatives of the flow angle with respect to the pressure values must be obtained using the chain rule as follows:

$$ \frac{\partial \alpha}{\partial P_i} = \frac{\partial \alpha}{\partial C_\alpha} \frac{\partial C_\alpha}{\partial P_i} \quad i = 1, 2, 3 $$

(3)

Then, equation (2) can be rewritten as:

$$ I_\alpha^2 = \left( \frac{\partial \alpha}{\partial C_\alpha} \right)^2 I_{C_\alpha}^2 $$

(4)

where:

$$ I_{C_\alpha}^2 = \sum_{i=1}^{3} \left( \frac{\partial C_\alpha}{\partial P_i} \right)^2 I_{P_i}^2 $$

(5)

Since $C_\alpha$ is a one-variable function (it depends only on the flow angle), partial derivatives are really total derivatives, and:

$$ \frac{\partial \alpha}{\partial C_\alpha} = \frac{1}{\partial \alpha} $$

(6)

Resolving $\partial C_\alpha/\partial \alpha$ for the relation of pressure combinations (1), it is obtained, after some algebra, that the uncertainty of the flow angle is given by:

$$ I_\alpha = I_p \cdot \sqrt{\left( C_{p3} - C_{p2} \right)^2 + \left( C_{p1} - C_{p3} \right)^2 + \left( C_{p2} - C_{p1} \right)^2} $$

$$ C_{p1} \left( C_{p3} - C_{p2} \right) + C_{p2} \left( C_{p1} - C_{p3} \right) + C_{p3} \left( C_{p2} - C_{p1} \right) $$

(7)

where it is assumed that the uncertainty of the pressure measured by the transducers, denoted as $I_P$, is the same for the three holes.

A similar procedure can be applied for the uncertainty levels of both dynamic and static pressures, resulting in:

$$ I_{P_d} = I_p \cdot \sqrt{\left( C_{p3} - C_{p2} \right)^2 + \left( C_{p1} - C_{p3} \right)^2 + \left( C_{p2} - C_{p1} \right)^2} $$

$$ C_{p1} \left( C_{p3} - C_{p2} \right) + C_{p2} \left( C_{p1} - C_{p3} \right) + C_{p3} \left( C_{p2} - C_{p1} \right) $$

(8)

$$ I_{P_s} = I_p \cdot \sqrt{\left( C_{p2} C_{p3} - C_{p3} C_{p2} \right)^2 + \left( C_{p3} C_{p1} - C_{p1} C_{p3} \right)^2 + \left( C_{p1} C_{p2} - C_{p2} C_{p1} \right)^2} $$

$$ C_{p1} \left( C_{p3} - C_{p2} \right) + C_{p2} \left( C_{p1} - C_{p3} \right) + C_{p3} \left( C_{p2} - C_{p1} \right) $$

(9)

The uncertainties of the flow angle and dynamic and static pressures are different for every specific probe, and it has been found that they are independent on the data reduction method: they are exclusively calculated from the pressure coefficients obtained during the calibration of the probe.

Figure 4 shows the uncertainties over the whole range of a cylindrical probe with a construction angle of 30 deg. The uncertainty of the flow angle is expressed as a percentage of the uncertainty in the pressure measurement, $I_p$, relative to the dynamic pressure $P_d$, while both dynamic and static pressures uncertainties are only referred to $I_p$, as befit the equations (7)-(9). When $\alpha=0$ deg, the uncertainty in the flow angle is about 0.15 deg for every 1% of $I_p/P_d$. This uncertainty level is practically maintained for all the angles ranged between $\pm 30$ deg. Outside this range, the levels increase progressively, reaching up to 0.5 when $\alpha=\pm 75$ deg. The uncertainty of the static pressure follows, basically, the same behaviour that the flow angle. The uncertainty of the dynamic...
pressure is maximum for zero-incidence flow ($\alpha=0$ deg) and minimum around $\pm30$ deg. Both dynamic and static pressures rise severely when operating in the limits of the attainable angular range of this probe ($\alpha=\pm75$ deg).

![Graph showing uncertainties of the flow angle and both dynamic and static pressures for a 30 deg cylindrical THP probe.]

Fig. 4: Uncertainties of the flow angle and both dynamic and static pressures for a 30 deg cylindrical THP probe.

As a practical application, the specific-designed cylindrical THP probe shown in figure 1 has been used to measure the flow rotor downstream of a single-staged, low-speed axial fan with a typical stator-rotor configuration. This dynamic probe has three miniature pressure sensors (Kulite CCQ-093 model) placed axially inside the probe, at 6 mm from the holes openings. The construction angle is 60 deg, and the diameter of the probe is 8 mm with a total length of 75 cm. The holes are placed 16 mm from the semi-spherical tip, with a 0.8 mm hole diameter.

Since the wake fluid presents an important velocity deficit associated to the blade blockage, there are important variations in both magnitude and direction of the flow in every blade passing period. Figure 5 shows the results of an instantaneous measurement. The pressure signals acquired in the holes have been already transformed to values of flow angle. Despite of the extremely turbulent signals, it is possible to identify the rotor wakes in the angle signal, with large variations ranging from -60 deg to 35 deg. Obviously, this justifies the election of a probe calibrated with a zone-based method that provides an extended angular resolution.

![Graph showing an instantaneous measurement of the flow angle with a 60 deg cylindrical THP probe.]

Fig. 5: Instantaneous measurement of the flow angle with a 60 deg cylindrical THP probe.

Finally, the influence of the probe construction angle over the limits of the angular range (in terms of double points and duplicated zones) was studied. It was found that the maximum angular range for a cylindrical probe is obtained for a construction angle of 35 deg (not shown here). To corroborate this conclusion, three different zones within the flow angles interval were considered: zone 1 when $P_1>P_2, P_3$; zone 2 when $P_2>P_1, P_3$; and zone 3 when $P_3>P_1, P_2$. For each zone, the following angular coefficients were defined:
The angular coefficient introduced in (10), compared to the traditional one, has been represented in figure 6 in the case of a cylindrical probe with a construction angle of 35 deg. As a result of this particular election, the zone-based angular coefficient is discontinuous, but it is possible to obtain the flow angle all over the valid angular range in a univocal way. Effectively, as shown in the figure, a 35 deg cylindrical THP probe, calibrated with a zone-based method that applies the angular coefficient defined in (10), provides an extended angular range close to ±90 deg.

![Graph showing traditional and zone-based angular coefficients for a cylindrical THP probe with a construction angle of 35 deg.](image)

**Fig. 6: Traditional and zone-based angular coefficients for a cylindrical THP probe with a construction angle of 35 deg.**

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**REFERENCES**


