TURBOMACHINERY BLADES SPECTRAL ESTIMATION FROM TIP-TIMING DATA

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ABSTRACT

In turbomachine industry, bladed assembly vibration measurements are very important for blades behaviour study. These measurements are generally obtained from strain gauges. However, one of the most promising methods for the analysis of blade vibrations in rotating bladed assemblies is the Blade Tip Timing or Optical Blade Vibration Measurement method.

A set of optical sensors is mounted on an engine casing, around a disc, and measures the times of arrival of each blade. These timings are then used to estimate the vibrations of the blades. However the fundamental problem for spectral analysis of blade tip-timing data is that the signals are undersampled and aliased. We propose here a new method for spectral estimation of blades responses from tip timing data that overcome these difficulties.

The method proposed in this communication is based on the use of a minimum variance filter associated with an iterative updating of the autocorrelation matrix. That allows to process correctly a signal even if the number of known signal samples is less than equivalent Nyquist criterion. This approach is suitable for spectral analysis of undersampled and aliased signals.

Performances of the spectral estimator have been evaluated for a mistuned rotor with aeroelastic instability. Tip timing data have been produced by a numerical model described in the text. The response spectrum is well identified for all blades. The method seems very promising for the monitoring of mistuned bladed discs.

INTRODUCTION

The Tip-Timing technology consists in the use of sensors which are mounted on an engine casing in front of a disc. They measure the times of arrival of each blade. Then these timings are used to estimate the vibratory motions.

This measure is cheaper than strain gauges, but its analysis is more difficult. Each sensor collects one time sample per revolution and per blade. For a blade which can have several oscillations per revolution, this sampling is very weak. Traditional Fourier methods are then useless to study this kind of signal. So tip-timing authors developed methods of analysis based on mechanical assumptions.

For each nature of vibratory response (asynchronous or synchronous), a strategy was adopted. For example, the study of synchronous vibration is based on the determination of the Engine Order which excites a mode during an acceleration (or deceleration) [1]. Good results are obtained if bladed disc modes are clearly distinct in frequency. Asynchronous vibration is generally treated by an All-Blade-Spectrum [2]. It is designed for a perfectly tuned flutter mode: all the blades vibrate at the same frequency and with the same amplitude. It only works well on simulated and real tip-timing data if the bladed disc respect this assumption.

We prefer to choose a different way. Our main goal is to study asynchronous vibrations in the case of instationary phenomena, without making assumptions on the mechanical behaviour of the system. So the spectral estimation was retained as the tool of identification for tip-timing data.

In this paper we propose a method based on the minimization of the variance of output signal for spectral estimation of aliased, non-uniform and irregular data. With this method it is not necessary to make hypothesis about the nature of the observed phenomena by tip-timing sensors.

We will see in a first part the tip-timing measure. Several remarks about the sampling's nature of the signals will be detailed in order to clearly understand the interest of a spectral estimator described in a second part. In a third part, a bladed disc simulator will be described. It aims to generate signals close to those obtained from experiments. Finally, the spectral estimator will be used on simulated signals for a particular case of mistuned bladed disc.

TIP-TIMING MEASURE

Sensors are placed in an engine's casing in front of a disc (figure 1).They can be optical, capacitive or eddy-current, but optical sensors are preferred because they have a better timing resolution. Another sensor is mounted in front of the shaft and only takes one information. The data provided by this sensor are then used to estimate the rotor speed.

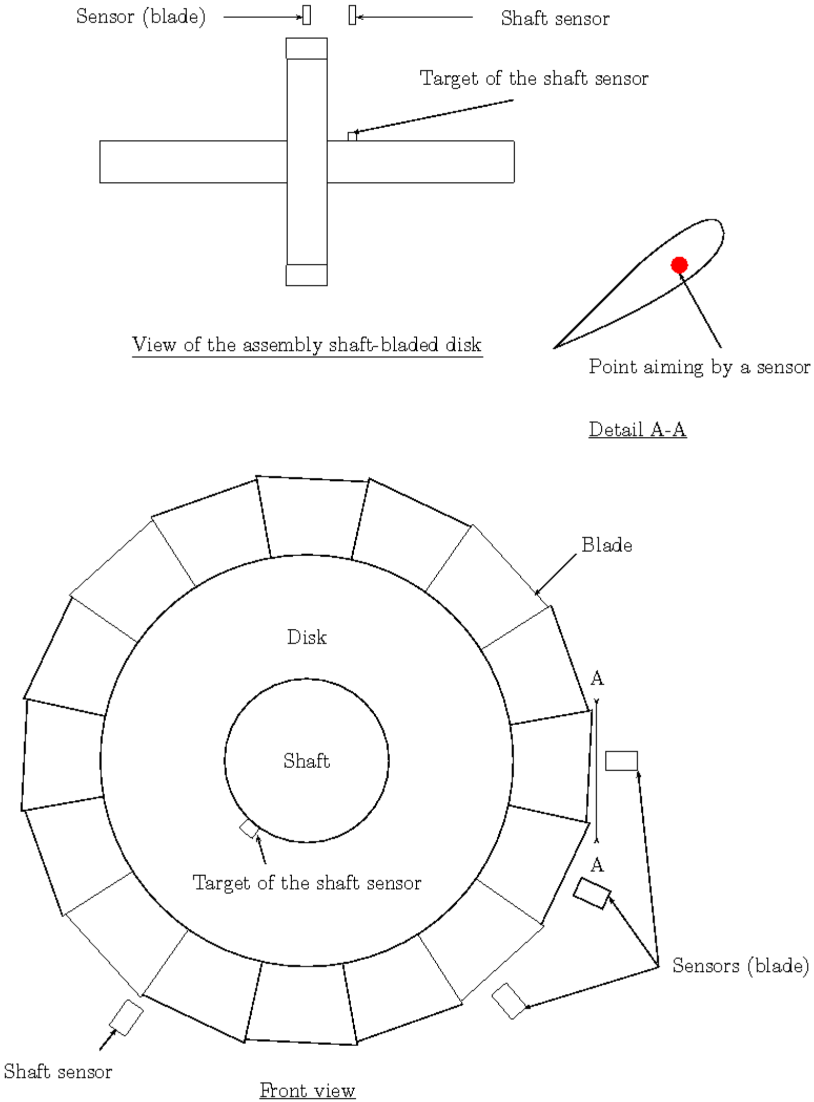

Figure 1 : Cyclic model

Each sensor measures the times of arrival of each blade (figure 2).If there were no vibration, the times of arrival of each blade in front of each sensor are regular and can be predicted. But blades vibrate, and their vibratory displacements change these times (figure 3). The differences between the measured times and the estimated times of arrival without vibration depend on the angular displacements of the blades, and so to the vibratory motions [1]. So vibratory displacements of blades can be estimated by tip-timing data. This fact will be explained in this section.

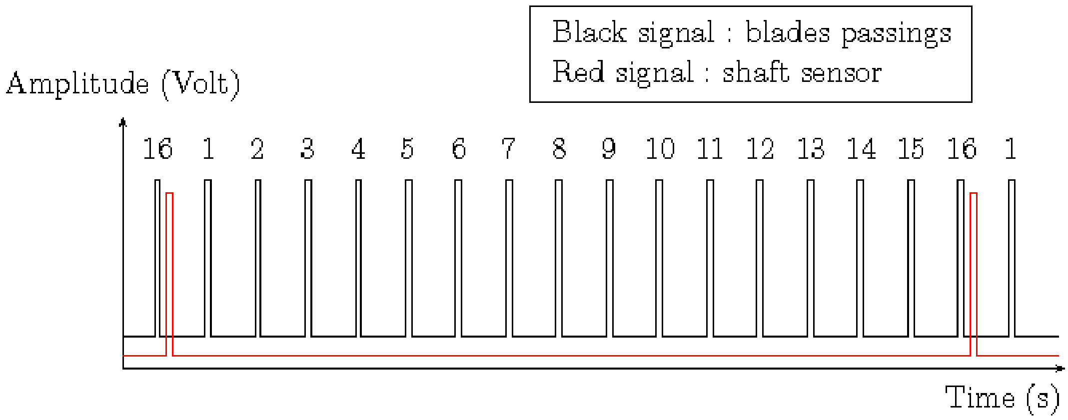

Figure 2 : Sensor signal

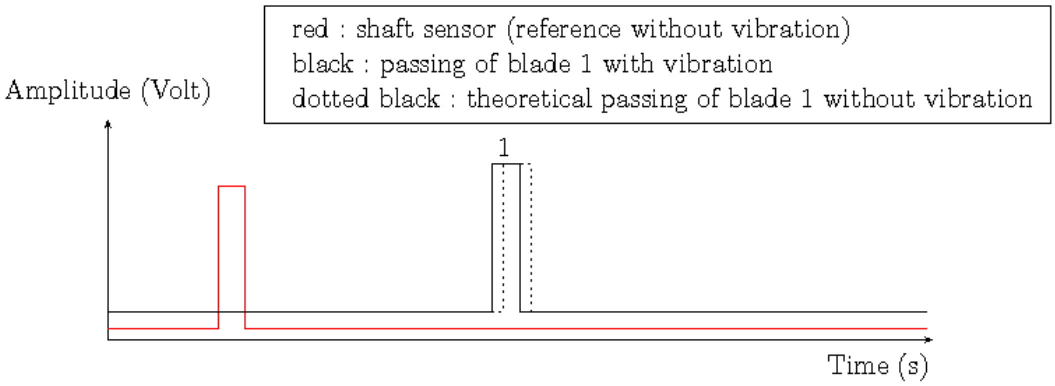

Figure 3 : Influence of vibration on the time of arrival

Let $x_k(t)$ be the angular deflection of the blade k and $\alpha_k(t)$ its angular position. They are linked by the relation

$$\alpha_k(t) = \int_0^t 2\pi F_r(u)du + x_k(t)$$

with F_r the rotation frequency. For the sake of simplicity, F_r should be assumed to depend linearly on the time:

$$F_r(t) = a * t + F_{r0}$$

with a the acceleration rate and F_{r0} the initial speed rotor. So the equation becomes

$$\alpha_k(t) = \alpha_{k0} + 2\pi(\frac{a}{2}t^2 + F_{r0}t) + x_k(t)$$

with α_{k0} the initial angular position of the blade k.

Let t_{kns} be the time of arrival of the blade k in front of the probe s at the revolution n and θ_s the angular position of the probe s. The angular position $\alpha_k(t_{kns})$ is then given by

$$\alpha_k(t_{kns}) = \alpha_{k0} + 2\pi(\frac{a}{2} * t_{kns}^2 + F_{r0}t_{kns}) + x_k(t_{kns})$$

$$\alpha_k(t_{kns}) = (n-1)2\pi + \theta_s$$

The fundamental relation of tip-timing sampling is then obtained:

$$x_{kns}(t_{kns}) = (n-1)2\pi + \theta_s - \alpha_{k0} - 2\pi(\frac{a}{2} * t_{kns}^2 + F_{r0}t_{kns}).$$

Several remarks should be made about this equation. Firstly, the measured time t_{kns} depends on the deflection x_{kns} , so the sampling time is a function of the observed object. It is called an implicit sampling.
Furthermore, the blade's deflection is the sum of the static and the dynamical displacements. The static displacement is due to centrifugal and temperature effects. The dynamical displacement is the vibratory displacement of the observed blade.

If we suppose that the excitation force contains a random part due to noise, then the blade's dynamical displacement (and the deflection x_k) contains a random part too. As seen previously, the sampling depends on the deflection, so if the deflection is random, then the sampling is random too.

Another point is that sampling is not uniform. Of course, sensors can be equally spaced around the casing, and if the rotor speed is kept constant, sampling can be supposed uniform. But on a test bench, it is very lucky if all sensors work well, and we have to suppose that one sensor can have a failure; then a spectral estimator based on regular sampling is irrelevant.

For a blade, the entire signal is formed from samples of all sensors. For example, on the figure 1, a blade signal is composed of three samples per revolution. Often, they are not equally spaced in the casing, so a true Nyquist frequency cannot be defined. But an equivalent "Nyquist frequency" f_{Ny} in the case of irregular sampling [3] can be defined by

$$f_{Ny} = \frac{N_s F_r}{2},$$

with N_s the number of sensors. Of course it is not the true mathematical Nyquist frequency, but it can give an idea of the undersampling of tip-timing data. Unfortunately, f_{Ny} is often very little compared to frequency range of interest. Data are then always undersampled. It is the main drawback of this technology.

SPECTRAL ESTIMATOR

Vibratory displacements can be estimated by the use of the times of arrival. Unfortunately, signals are severely undersampled. In addition, the sampling is irregular. In this section, the feasibility to study these signals is discussed and the choice of the spectral estimator is explained.

For a real bandwidth signal $x(t)$ whose maximal frequency is f_{max}, let $X(f)$ be its Fourier transform. The minimal sampling rate is $2f_{max}$ (Nyquist theorem). A subset of this class is composed by multiband signals. They are distinguished by their spectral supports F_s, defined as the set of frequencies over which the spectrum $X(f)$ of the signal is nonzero. On the figure 4, $\chi(f)$ is the existence function of the Fourier transform $X(f)$of a multiband signal. $\chi(f)$ is defined by:

$$\chi(f) = 0 \text{ if } X(f) = 0$$
$$\chi(f) = 1 \text{ if } X(f) \neq 0.$$

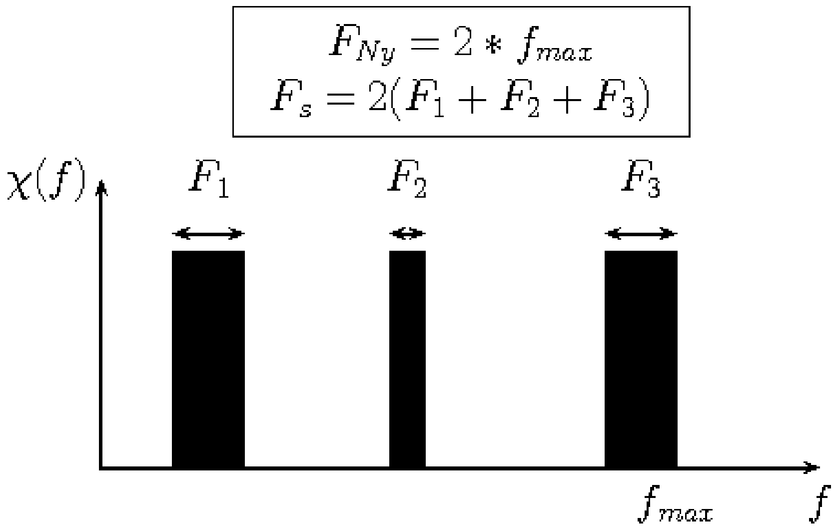

Figure 4 : Frequential support of a multiband signal

F_s can contain several bands in the frequency domain. Landau showed ([4]) that the sampling rate of an irregular sampling for the class of multiband signals is lower-bounded by F_{La}, defined by:

$$F_{La} = \sum_{p=1}^{P} F_p$$

with P the number of frequency bands and Fp the width of a frequency band. In practice, F_{La} could be significantly smaller than the Nyquist rate. In our case, F_s is unknown. But a bladed disc behaves like an oscillator and filter frequency bands, generally limited by the structural modes. So an estimator which can take advantage of irregular sampling should tend to the minimal sampling rate F_{La} and should be relevant to analyse multiband signals obtained by the tip-timing technology.

Experimental signals often contain a large part or randomness. Although they could not be periodic, spectral analysis can still be made by the use of the autocorrelation function. This probabilistic tool measures the linear dependence of a process with itself during time and is directly linked to the Fourier transform of a random signal.

Several papers aim to estimate spectra for irregular and aliased data [5-7]. A few of them are based on the estimate of the autocorrelation function [8,9]. Among them, one was chosen because of its performance and its relative simplicity for non-specialists in spectral estimation.

This method is presented by Greitans [9-11]. It belongs to Capon's family of spectral estimators and is based on properties of nonuniform sampling, filtering by minimization of the process output and an iterative algorithm on autocorrelation matrix. The main idea is to minimize the variance of the narrowband filter output signal. The frequency response of the filter adapts itself to the input signal spectral components on each frequency of interest. The variance of the output process is determined by $\rho=a^H Ra$ (see [12]), with a the vector of coefficients filter, $(.)^H$ the Hermitian and R the signal autocorrelation matrix. Coefficients have to verify that, on each frequency f_0, the gain of the filter response is one: $e^H(f_0)a=1$, with $e_i(f_0)=exp(j2\pi f_0 t_i)$.. It means that a sinusoid at frequency f_0 passes through the filter designed for it without distortion.

Filter's coefficients are given by:

$$a(f_0)=\frac{R^{-1}e(f_0)}{e^H(f_0)R^{-1}e(f_0)}.$$

The spectral amplitude is obtained by $s(f_0)=xa(f_0)$, with x the nonuniform undersampled signal.

According to the previous equation , filter's coefficients depend on the signal autocorrelation matrix. The usual way to estimate is based on the average of the mutual products of signal samples. If the sampling is not uniform, this approach is irrelevant. Greitans proposed to use the Wiener-Khintchin theorem which links the autocorrelation function and the power spectral density (PSD):

$$r(\tau)=\int_{-\infty}^{\infty}P(f)e^{j2\pi f\tau}df\ ,$$

with $P(f)$ the PSD of the signal. The easiest way to obtain a first approximation of the PSD is to use a nonuniform discrete Fourier transform (NFFT):

$$P(f)=\frac{1}{N^2}\left|\sum_{k=1}^{N}x_k e^{-j2\pi f t_k}\right|^2.$$

Values of the autocorrelation signal matrix R computed by this first approximation of $P(f)$ are false by artifacts. An iterative algorithm described by Liepin'sh [13] improves results. The (i+1)th order estimate of signal autocorrelation matrix is updated from the (i)th order estimate $\hat{P}^i$:

$$\hat{R}_{lk}^{(i+1)}=\sum_{m=1}^{M}\hat{P}_m^{(i)}\exp(j2\pi f_m(t_k-t_l)).$$

Then, using $\hat{R}^{(i+1)}$, the estimates $\hat{S}^{(i+1)}$ and $\hat{P}^{(i+1)}$ are calculated by

$$\hat{P}^{(i+1)}=\left|\frac{E(\hat{R}^{(i+1)})^{-1}x^T}{diag(E(\hat{R}^{(i+1)})^{-1}E^H)}\right|^2,$$

with $E_{mn}=exp(-j2\pi f_m t_n)$.

This algorithm is very slow due to the nonuniform discrete Fourier transform used during the formation of the estimate autocorrelation matrix $\hat{R}$. As a Fast Fourier Transform can not be used, the computational complexity is $O(N^2M)$. Approximate Fast Fourier Transforms are used to improve the speed [14] and the computational complexity reduces to $O(N^2*log(N)+log(1/\varepsilon)M)$, with ε the desired accuracy.

We choose a convergence's criterion based on the PSD:

$$Error=\left|\frac{\hat{P}^{(i+1)}-\hat{P}^{(i)}}{\hat{P}^{(i)}}\right|*100.$$

The algorithm is stopped when the error is small enough. In practice, Error=5 was chosen. If the algorithm does not converge, more sensors or data are required.

AEROMECHANICAL MODEL

Simulator's goal is to produce data close to those obtained on a test bench. It calculates the times of arrival of blades in front of sensors. Contrary to models previously made in the study of tip-timing [15,16], this one aims to be able to simulate mistuned bladed discs and flutter behaviour.

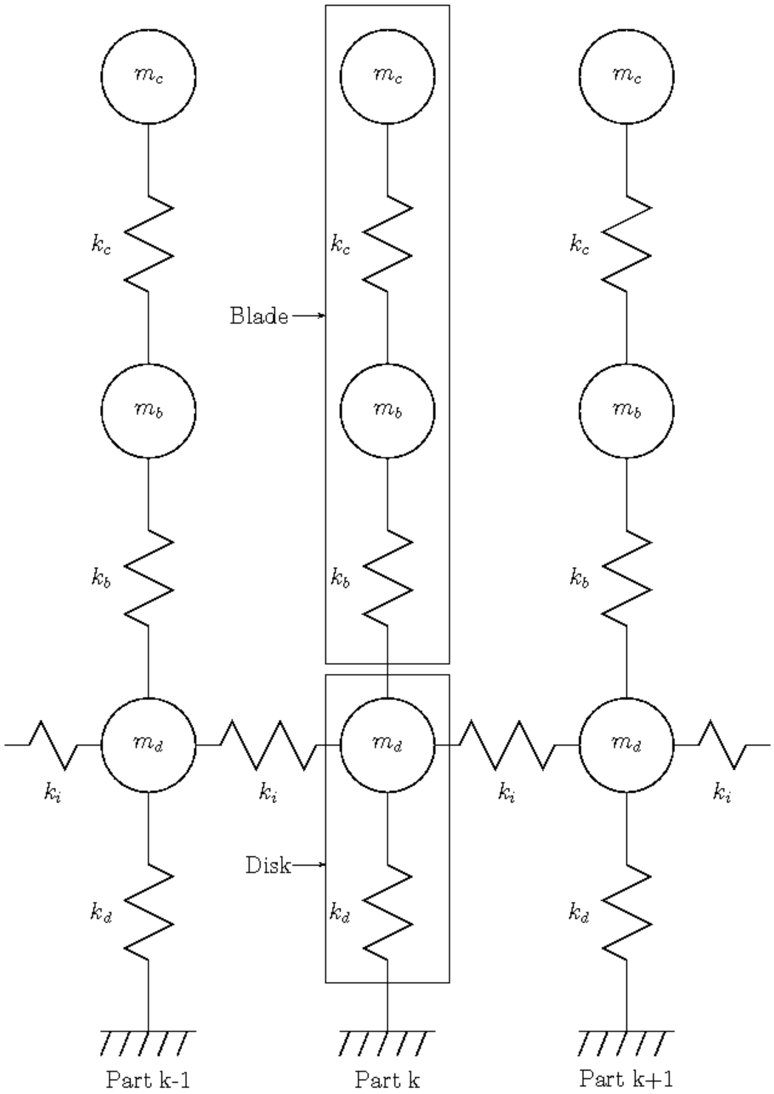

Figure 5 : Cyclic model

A bladed disc is splitted into N sectors, with N the number of blades (figure 5). Each sector is constituted by one degree of freedom (dof) for the disc's part and two dofs for the blade. The disc's coupling is represented by the springs k_i. When a bladed disc is manufactured, some variations of dimensions appear. The symmetry can not be perfect. Random variations are introduced in the springs k_b and k_c to modelize it.

The displacement vector for the whole assembly is $x=[x_1\ x_2\ \dots\ x_N]^T$, with x_i the displacement vector for the sector i. The Lagrange's method gives the matrix equations:

$$M\ddot{x} + (C + C_a(\Omega))\dot{x} + K(\Omega)x = F(t)$$

with M the mass matrix, C the damping matrix, C_a the aerodynamic matrix, K the stiffness matrix, $F(t)$ the excitation force vector and Ω the rotor speed. Equations are integrated in the time domain in order to obtain the temporal responses.

The system can become unstable because of energy transfer between fluid and structure: it is the flutter phenomenon. For real machines, the vibratory amplitude grows until collapse or until a limit cycle is reached due to non linear effects. For a detailed review of aeroelastic methods, see the paper of Marshall [17].

Here we use a simple model: fluid-structure coupling is represented by an aerodynamic matrix C_a multiplied by the speed vector $\dot{x}$. The N second-order differential equations are reduced to $2N$ first-order differential equations. A first-order state-space model is then obtained:

$$\begin{bmatrix} C+C_a & M \\ M & 0 \end{bmatrix}\begin{pmatrix} \dot{x} \\ \ddot{x} \end{pmatrix} = \begin{bmatrix} -K & 0 \\ 0 & M \end{bmatrix}\begin{pmatrix} x \\ \dot{x} \end{pmatrix} + \begin{pmatrix} F(t) \\ 0 \end{pmatrix}$$

The stability of the system is then studied by evaluating the generalized eigenvalues of the system. The imaginary part of each eigenvalue is related to the frequency of the aeroelastic mode and the real part to the damping. If a real part is strictly positive, the considered mode is unstable.

Fluid goes upstream through a number of obstacles N_{ob} and takes a spatial shape of wakes. So each blade is harmonically excited by a travelling wave:

$$F(t) = \sum_{k=1}^{n+1} A_k \cos(kN_{ob}\Omega t) + B_k \sin(kN_{ob}\Omega t)$$

with n the number of harmonics. In our case, we choose $B_k=0$ for all k and n=2 to simplify the model.

RESULTS AND DISCUSSION

Flutter is an asynchronous vibration and is generally treated by an all-blade spectrum method [2]. This method supposes that every blade vibrates at the same frequency and at the same amplitude. It works well for a quasi-perfectly tuned disc, but it is not suitable if mistuning causes vibratory energy localizations. Mistuning is sometimes used by industrials to suppress cases of flutter and unstable mistuned rotor analysis cannot be found in the literature.

The simulated assembly has 21 blades. The mistuning pattern is shown on the figure 6. We assume that its diameter is 440 mm and the rotor speed is 18600 rpm. The acceleration is weak and constant: 60 rpm/min. 4 sensors are disposed around the casing. Their positions are randomly chosen. The equivalent Nyquist frequency is 620 Hz.

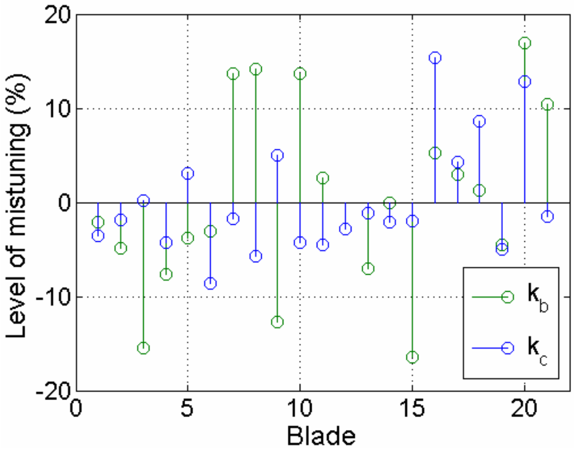

Figure 6 : Mistuning pattern of the rotor

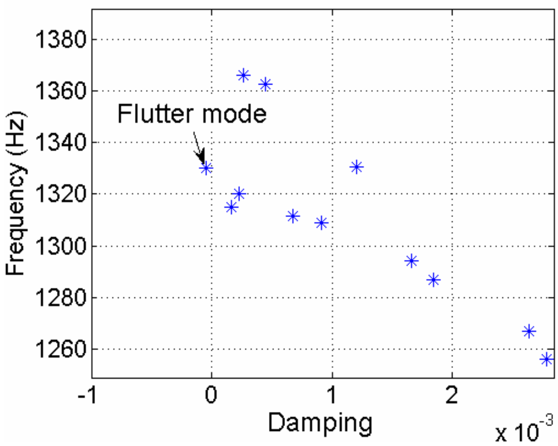

Figure 7 : Stability diagram

The frequency range of interest is between 0 and 5 000 Hz. A synchronous excitation is considered: a 6th Engine Order. It does not coincide to any eigenfrequencies. Furthermore the system is aerodynamically unstable (flutter). A part of the initial stability diagram is shown on the figure 7. On this figure, each mode is represented as a function of its damping and its frequency. Modes with positive damping are stable. They form the major part of the set. But one mode at 1331 Hz has a negative damping: it is the flutter mode. It is characterized by a exponentially growing part. In this example, the flutter damping is weak: then during some seconds, the vibratory displacement can linearly grow.

The mode shape of the flutter mode is shown on the figure 8. This mode is strongly localized on the blades 6-10.

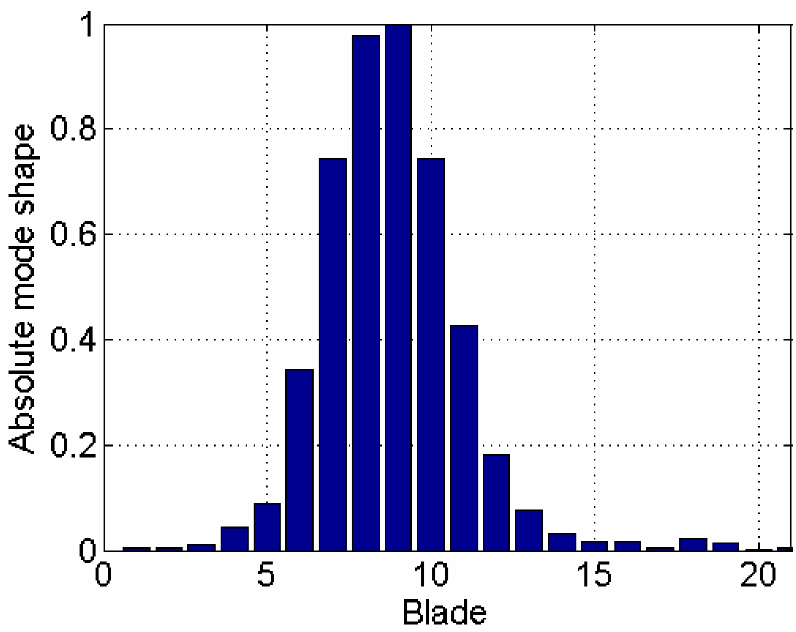

Figure 8 : Mode shape of the flutter mode

The temporal response of the blade 9 is shown on the figure 9. The flutter is weak, so the vibration slowly grows, as previously explained. Three seconds are simulated. At the beginning, the blade 9 amplitude is 50 µm. After 3 seconds, the amplitude was doubled. The simulation ends then, because in a real case, no-linear effects can stabilize the growing. Our model can not simulate it. In the worst case, the blade continues to exponentially growing until collapse.

In practice, tip-timing data contain noise due to measurements uncertainties. We simulate it by random noise put on the times of arrival of the blades in front of the sensor. The noise power is chosen carefully. The theoretical precision was 5 µm by our measurement technology. So the value of the noise amplitude was chosen as 10 µm in order to be coherent. The simulated displacement on the blade seen by the sensor at 0° is shown on the figure 10. As other spectral estimators, the presented method depends on the Signal-on-Noise Ratio (SNR). The spectral estimator should then be more efficient for the blades which more than the others, like the blades 8-10. That's why the estimator will be firstly tested on the blade 9.

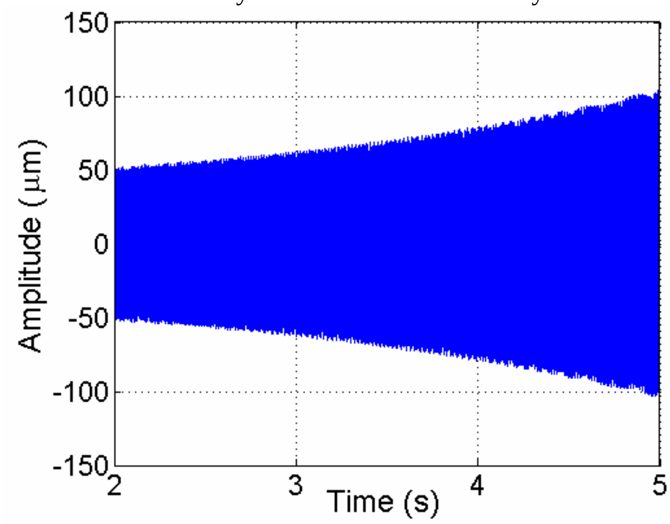

Figure 9 : Temporal response of the blade 9

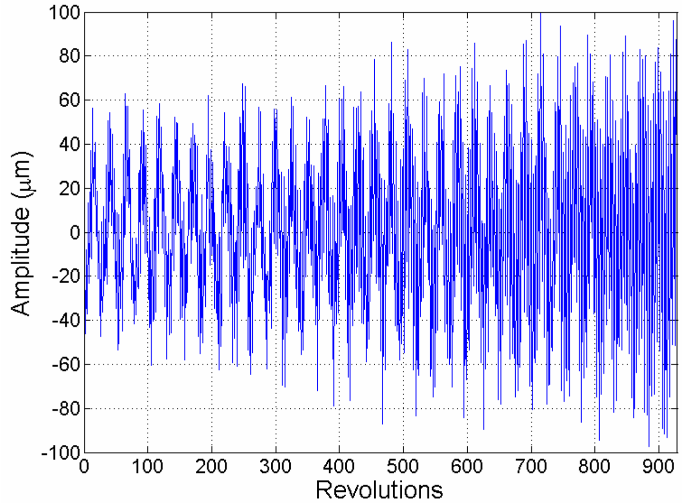

Figure 10 : Vibratory displacement of the blade 9 seen by a tip-timing sensor

The proposed method was tested on 30 revolutions (800-830). The figure 11 shows the first obtained spectrum by a nonuniform Fourier transform. It seems to contain a lot of modes. The major part of them is replicas due to the severe undersampling because the equivalent Nyquist frequency is 620 Hz and the frequency range of interest is 0-5000Hz. This spectrum cannot be studied directly without previous knowledge of true modes. Furthermore, calculated spectral amplitudes can be expected to be biased.

The algorithm gives the spectrum shown on the figure 12. One mode can easily be identified at 1331 Hz

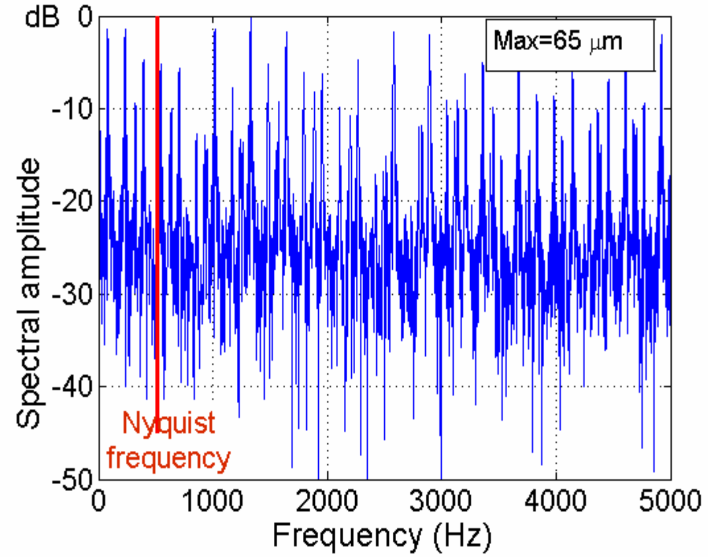

Figure 11 : Initial spectrum of the blade 9

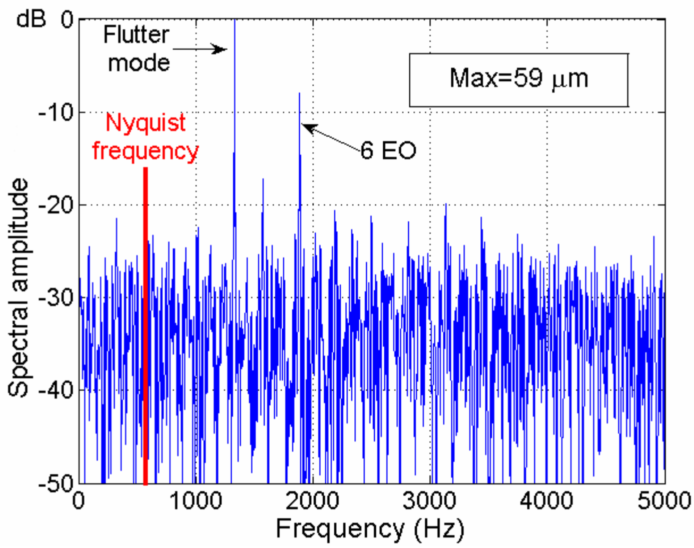

Figure 12 : Final spectrum of the blade 9

after ten iterations, for an error=2%. It's the flutter mode. The precision in frequency is 2.6 Hz. The 6 EO can be seen too at 1860 Hz. Amplitudes of artefacts due to undersampling disappear.

Spectra are estimated on all blades and the flutter mode can be precisely studied (figure 13). The measured response on the figure is composed by the absolute amplitude at 1331 Hz of the spectra obtained for each blade. As expecting by the simulation, the mode is strongly influenced by the mistuning. The major part of the vibratory energy is contained in the blades 6-11. There is a good correlation with the theoretical flutter mode, but it is not perfect because of the noise in measured signals.

Another remark is that the convergence of the method depends of the studied blade. The convergence is easily reached for the blades 6-11, but not for the others. The amplitudes of the other blades are very weak and so the noise is powerful compared to the flutter amplitudes. The spectral estimator is designed for a multiband signal. So, if a signal does not show a multiband structure, this spectral estimator should not be suitable.

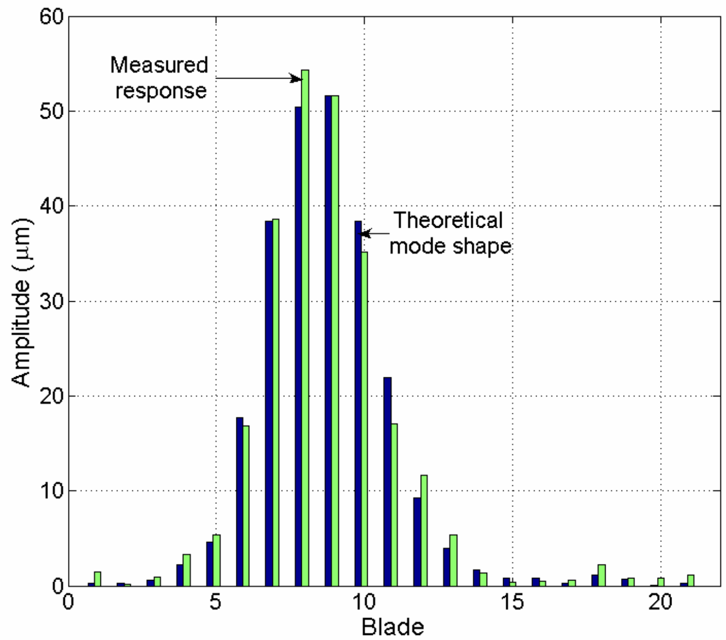

Figure 13 : Theoretical mode shape and measured response

Nevertheless, we can identify by this spectral estimator localizations of energy in mistuned bladed assemblies for a phenomenon like mistuned flutter. The whole assembly can be studied, blade per blade: this is a great advantage compared to strain gauges which only monitor one or two blades.

CONCLUSION

In this paper we propose a new method in order to analyse tip-timing data. A bladed disc simulator is described. It can simulate aeroelastic effects like flutter and provide tip-timing data close to those obtained on test rigs.

The main features of tip-timing measurements have been explained with an emphasis on the undersampled and irregular nature of the data. A spectral estimator designed for this particular sampling gives good results on simulated data of mistuned flutter. The frequency range of interest of estimated spectra can be several times the equivalent Nyquist frequency.

By this method, the response level of each blade can be obtained. Contrary to methods usually used for tip-timing data analysis, no assumptions about mechanical behaviour of bladed discs were made in order to obtain a parameter estimator. Hence this method can analyse any phenomena, including forced response and instability for tuned and mistuned rotors.

In the future, we will try to study the bias of amplitudes computed by this spectral estimator and to compare them with other measurements in order to completely validate the methodology.

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