2D DATA REDUCTION TECHNIQUE FOR TURBINE BLADE HEAT TRANSFER MEASUREMENTS USING DOUBLE-LAYERED THIN FILM GAUGES

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ABSTRACT
This work presents a novel technique to post-process heat transfer measurements performed with double-layered thin film gauges. It consists of solving the two-dimensional unsteady heat conduction equation in a double-layered cross sectional area of a turbine airfoil. Unlike the common 1D approach, this extended methodology is capable to capture the lateral heat conduction phenomenon that mainly appears in the leading edge of the blade. Moreover, the semi-infinite substrate hypothesis is not anymore employed. This is specially useful to compute the wall heat flux in the trailing edge of the blade.

Two test cases have been presented. The first of them demonstrates the existence of local 2D effects in the cross sectional area of a rotor blade. The second test shows the potential of the 2D modelling for computing the initial temperature distribution in short duration heat transfer experiments.

INTRODUCTION
Thin film gauges are nickel resistive detectors which are usually employed in short duration facilities. They provide the time dependent surface temperature evolution of an instrumented airfoil during blow-down tests. This information allows computing the convective heat transfer coefficient around the airfoil and subsequently, the Nusselt number distribution. This is specially useful for the design of cooling schemes in turbine blades, and for the determination of the boundary layer status of the tested airfoil.

Single-layered and double-layered substrates constitute the technical solutions usually employed to implement thin film gauges onto airfoils in short duration facilities. The single-layered thin-film gauges used at VKI consist of platinum thin-film gauges fired onto a machinable glass ceramic substrate. The entire blade can be made of ceramic for cascade testing (Arts et al. [1]) or inserts can be fitted into metallic blades in the case of measurements on the rotor of a turbine stage (Didier et al. [2] and Dénos [3]). Even in regions with high curvature, like the leading edge of an airfoil, data reduction of single-layered thin film gauges relays on a 1D approach. This is due to the low thermal conductivity of the ceramic insert and the short duration of the test. Moreover, the thickness of the ceramic insert justifies the assumption of semi-inifinite substrate during the test duration. (Dénos [3]).

The double-layered thin film gauge technique was intensively developed at the University of Oxford (Doorly and Oldfield [4]). In the applications that are currently carried out at VKI (Iliopoulou et al. [5]), the sensing element is deposited on a polyamide sheet wrapped around the airfoil with a double-sided adhesive. They provide a faster and more flexible implementation than the ceramic inserts. However, the data reduction is more complex, as the heat conduction equation has to be solved in a double layered substrate, whose thermal properties must be determined.

In previous works (Billiard et al. [6]), the data reduction of double-layered thin film gauges relied as well on the assumptions of 1D heat conduction and semi-infinite substrate. In the present work, the suitability of these hypotheses is questioned. A new approach for the data reduction of double-layered thin film gauges is proposed. It consists of the solution of the 2D unsteady heat conduction equation in the cross sectional area of the tested body.

NOMENCLATURE
C	specific heat at constant pressure [*J/(kg K)*]
h	heat transfer coefficient [*W/(m²K)*]
κ	thermal conductivity [*W/(m K)*]
L	layer thickness [m]
Nu	Nusselt number [hD/κ]
$\dot{Q}$	heat flux [W/m²]

T temperature [K]
t time [s]
n normal to the wall
N interpolation function
K stiffness matrix
A $\frac{\sqrt{\rho_1 C_1 \kappa_1}-\sqrt{\rho_2 C_2 \kappa_2}}{\sqrt{\rho_1 C_1 \kappa_1}+\sqrt{\rho_2 C_2 \kappa_2}}$

Greek

α thermal diffusivity [m^2/s]
ρ density [kg/m^3]
Ω numerical domain

Subscripts

i triangular element nodes
e finite element
0 initial condition
f final condition
1 index for the first layer
2 index for the second layer
n index for time step/counter
∞ semi-infinite
W wall

DETERMINATION OF SURFACE HEAT FLUX FROM THIN FILM GAUGES MEASUREMENTS

The determination of the local convective heat transfer coefficient requires the knowledge of 1) the reference gas temperature 2) the local wall temperature 3) the heat flux to the instrumented wall blade in the measurement location:

$$h=\frac{\dot{Q}_W(t)}{T_{gas}-T_W(t)} \tag{1}$$

The gas temperature T_{gas} is measured with the instrumentation available at the test rig (Paniagua et al. [7]), and the wall temperature evolution $T_W(t)$ is monitored by the thin film gauge. The heat flux is derived from the wall surface temperature history during a blow-down test: hot gas is suddenly released in the cold-instrumented test airfoil, and the surface temperature rises as a function of time.

The common approach for the computation of the wall heat flux relays on a one-dimensional hypothesis. Analog circuits can be built that mimic the transfer function linking the surface temperature increase to the wall heat flux (Doorly and Oldfield [8]). The numerical solution of the 1D unsteady heat conduction equation for a multilayered substrate, written as follows, is more flexible:

$$\frac{1}{\alpha}\frac{\partial T}{\partial t}=\frac{\partial^2 T}{\partial x^2} \tag{2}$$

The thermal properties are obviously depending on the considered location x inside the substrate. The thermal properties of the adhesive sheet are similar to the Upilex sheet, thus both are considered as a unique first layer. The second layer is the blade material itself.

An initial temperature distribution in the 1D domain is imposed. Two boundary conditions are necessary to solve Eq. (2): the temperature history provided by the thin film in the measurement location, and the semi-infinite substrate condition in the opposite bound (see Fig. 1).

$$\begin{array}{ll} T(x,\ 0)=T_0 & 0\le x\le L_\infty \\ T(0,t)=T_W(t) & 0<t\le t_f \\ T(L_\infty,t)=T_0 & 0<t\le t_f \end{array}$$

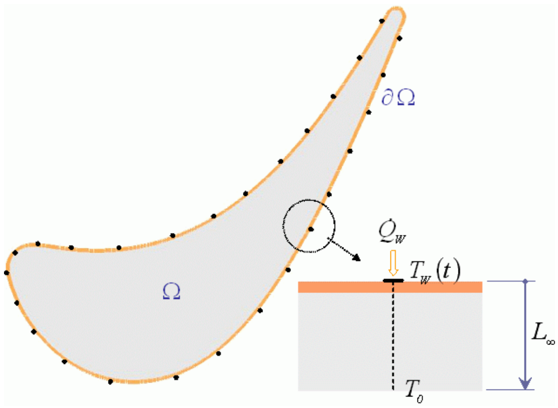


Fig. 1. Computational domain in the 1D approach

At the layers interface $x=L_1$, the continuity in heat flux must be satisfied at every time step:

$$-\kappa_1\left.\frac{\partial T(t)}{\partial x}\right|_{x=L_1^-}=-\kappa_2\left.\frac{\partial T(t)}{\partial x}\right|_{x=L_1^+} \tag{3}$$

Finally, the heat flux to the wall is computed with the Fourier law:

$$\dot{Q}_W(t)=-\kappa_1\left.\frac{\partial T(t)}{\partial x}\right|_{x=0} \tag{4}$$

A Crank-Nicholson discretization scheme is proposed by Dénos [3] for the solution of the 1D problem. The sensing element of the gauge is so thin and conductive that it can be neglected in the modeling (Schultz and Jones [9]).The first layer of the substrate must be an electrical insulator because of the sensing element and the connections painted onto its surface. In order to magnify the surface temperature measured by the thin film, the first layer is chosen to be a thermal insulator (low thermal diffusivity).

It is commonly attributed to this low thermal diffusivity the quality of minimizing lateral heat conduction, as done in ceramic substrate layouts. Under this assumption, together with the existence of semi-infinite substrate behaviour, the previous 1D approach is employed to post-process heat transfer measurements with double-layered thin film gauges.

Nevertheless, in certain regions of the cross-sectional area of a turbine airfoil, nor 1D heat conduction, neither semi-infinite hypothesis can be assumed. Heat transfer measurements performed with ceramic substrate thin film gauges, where the previous hypothesis are clearly satisfied, diverge from equivalent results of double-layered substrates in the leading and trailing edges.

The lateral heat conduction phenomena in the leading edge has been modelled by some authors considering the radial 1D unsteady heat conduction equation (Buttsworth and Jones [10]). However, a rigorous computation of the wall heat flux in the external boundary of the airfoil requires the extension of the problem to a two-dimensional approach.

The solution of the 2D unsteady heat conduction equation in the double-layered cross sectional area of the tested airfoil is proposed. The initial temperature distribution, together with the temperature history provided by the thin film array around the airfoil, are necessary to solve the problem:

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \tag{5}$$

$$T(x,y,0) = T_0(x,y) \qquad x,y \in \Omega$$
$$T(x,y,t) = T_W(x,y,t) \qquad x,y \in \partial\Omega \qquad 0 < t \le t_f$$

The normal heat flux to the wall of the airfoil is obtained with the fourier law:

$$\dot{Q}_W(x,y,t) = -k_1 \cdot \vec{n}\nabla T\,,\ x,y \in \partial\Omega \tag{6}$$

MATHEMATICAL MODEL

A weighted residual (Galerkin) approach is used to derive the finite element equations from the governing differential equation Eq. (5). The solution of the resulting algebraic system will allow obtaining the temperature distribution inside the body and, subsequently, the normal heat flux to its external boundary (Eq. (6)).

The double layered domain is divided into E finite elements of 3 nodes each. The interface between the upilex and the blade is forced to coincide with the corresponding elements faces. First order interpolation functions are chosen to define a suitable form of variation of the temperature T in each finite element *'e'*, as a function of the temperature in its 3 nodes, $\vec{T}^{(e)}$:

$$T^{(e)}(x,y,z,t) = N(x,y,z)\cdot\vec{T}^{(e)}(t) \tag{7}$$

The integral of the weighted residue over the domain of the element is set equal to zero by taking the weights same as the interpolation functions N_i:

$$\iint_{\Omega^{(e)}} N_i\left(\frac{\partial^2 T^{(e)}}{\partial x^2} + \frac{\partial^2 T^{(e)}}{\partial y^2} - \frac{1}{\alpha}\frac{\partial T^{(e)}}{\partial t}\right) d\Omega = 0,\ i = 1,2,3 \tag{8}$$

For the problem posed in Eq. (5), where the whole boundary is prescribed with a temperature distribution $T_W(x,y,t)$, Eq. (8) can be expressed in matrix form as (Rao [11]):

$$K_1^{(e)}\cdot\vec{T}^{(e)} + K_3^{(e)}\cdot\dot{\vec{T}}^{(e)} = 0 \tag{9}$$

The expressions $K_1^{(e)}$ and $K_3^{(e)}$ can be stated using matrix notation as

$$K_1^{(e)} = \iint_{\Omega^{(e)}} B^T D\, B\, d\Omega \tag{10}$$

$$K_3^{(e)} = \iint_{\Omega^{(e)}} \rho C N^T N\ d\Omega \tag{11}$$

where

$$D = \begin{bmatrix} \kappa & 0 \\ 0 & \kappa \end{bmatrix} \text{ and } B = \begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} & \frac{\partial N_3}{\partial x} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_2}{\partial y} & \frac{\partial N_3}{\partial y} \end{bmatrix}$$

The element equations Eq. (9) are assembled in the usual manner, and the temporal derivative term is discretized with a backward difference scheme (Stasa [12]):

$$K_1\cdot\vec{T}^{n+1} + K_3\cdot\frac{\vec{T}^{n+1}-\vec{T}^{n}}{\Delta t} = \vec{0} \tag{12}$$

The final linear system of algebraic equations is:

$$\left(K_1\cdot\Delta t + K_3\right)\cdot\vec{T}^{n+1} = K_3\cdot\vec{T}^{n} \tag{13}$$

The initial temperature distribution $\vec{T}^0$ is used to start the computation, and the boundary conditions at every time step are imposed by restricting the values of the corresponding boundary nodes.

The heat flux to the wall is computed in every wall elements as follows:

$$\dot{Q}_w^{(e)} = -\kappa_1\left(n_x\frac{\partial T^{(e)}}{\partial x} + n_y\frac{\partial T^{(e)}}{\partial y}\right) \tag{14}$$

where

$\vec{n} = (n_x, n_y)$ is the normal vector to the wall boundary in the element location, and the local temperature derivatives are computed with the classical finite element formulation:

$$\left.\begin{aligned} \frac{\partial T^{(e)}}{\partial x} &= \frac{\partial \vec{N}}{\partial x}\cdot\vec{T}^{(e)} \\ \frac{\partial T^{(e)}}{\partial y} &= \frac{\partial \vec{N}}{\partial y}\cdot\vec{T}^{(e)} \end{aligned}\right\} \tag{15}$$

NUMERICAL VALIDATION

Doorly and Oldfield [8] obtained the analytical temperature evolution in the external boundary of a 1D, semi-infinite double-layered substrate, linked to a wall heat flux step $\dot{Q}_W$:

$$T_W(t) = \frac{2\dot{Q}_W}{\sqrt{\rho_1 C_1 \kappa_1}} \left[(t/\pi)^{1/2} + 2\sum_{n=1}^{\infty} A^n \left\{ (t/\pi)^{1/2} exp\left[\frac{-n^2 L_1^2}{\alpha_1 t} \right] - \frac{nL_1}{\sqrt{\alpha_1}} erfc\left(nL_1 / (\alpha_1 t)^{1/2} \right) \right\} \right] \quad (16)$$

This expression is imposed as boundary condition of the unsteady 1D heat conduction equation, in a double-layered semi-infinite domain. Consequently, the wall heat flux reconstructed in every time step is $\dot{Q}_W$. This property is employed by Billiard et al. [6] to validate the finite-difference discretization schemes of the 1D unsteady heat conduction equation.

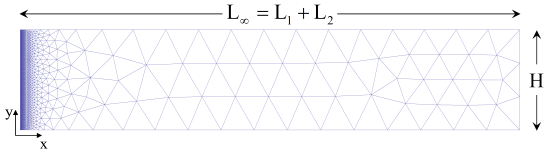


Fig. 2. Quasi-1D domain for numerical validation

The validation of the proposed 2D FEM model requires the construction of a two-dimensional, double-layered domain, as depicted in Fig. 2. The external layer (left side) represents the Upilex wrapped around the blade, and the internal layer is the blade material itself.

The dimensions of the domain assure that, under uniform boundary conditions, the temperature diffusion phenomenon is one-dimensional. The total length of the substrate is over the critical length of the substrate, computed as follows (Dénos [3]):

$$L_\infty = 3.648\sqrt{\alpha_1 \cdot t_f} \quad (17)$$

This very conservative expression assures that, for the test duration t_f, the assumption of semi-infinite substrate is satisfied.

The properties of the two layers that constitute the domain are shown in Table 1. These properties have been obtained with the calibration procedure assessed at VKI, for the stator blades instrumented in the CT3 facility (Iliopoulou et al. [5]).

Table 1. Properties of the double-layered domain

	L (mm)	ρ (kg/m^3)	C $(kJ/kg \cdot K)$	κ $(W/m \cdot K)$
1st layer	0.175	1470	1130	0.288
2nd layer	17.5	7900	460	18

The external layer has been meshed with triangular, structured elements, while the internal layer presents an unsctructured mesh. A careful sensitivity analysis has been carried out to assess the thickness of the elements adjacent to the external Upilex wall. Due to the high difference in thermal products of both layers ($\sqrt{\rho_1 C_1 \kappa_1} = 692 \;\; J/m^2Ks^{1/2}$, $\sqrt{\rho_2 C_2 \kappa_2} = 8088\, J/m^2Ks^{1/2}$), and the low relative thickness of the first layer, huge temperature gradients appear in the Upilex wall at every time step (see Incropera and DeWitt [13]). A wall element thickness of $1.8\,\mu m$ has proved to accurately capture the wall temperature gradients. Wall elements with a length 20 times bigger than their thickness permit to decrease the total size of the mesh without any loss of computation accuracy.

The next initial and boundary conditions have been imposed to solve the problem:

$$T(x,y,0) = 0^o C, \qquad 0 \le x \le L_\infty \qquad 0 \le y \le H$$
$$T(0,y,t) = T_W(t), \qquad 0 \le y \le H, \qquad 0 < t(s) \le 0.5$$
$$T(L,y,t) = 0^o C, \qquad 0 \le y \le H, \qquad 0 < t(s) \le 0.5$$

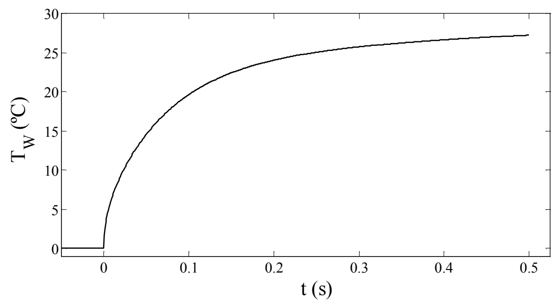


Fig. 3. Analytical wall temperature evolution for a 1D domain submitted to a constant wall heat flux

The temperature evolution $T_W(t)$, during the 0.5 seconds of the test duration, is sketched in Fig. 3. This evolution is built with Eq. (16) imposing the characteristic value of $\dot{Q}_W = 40000\, W/m^2$, and defining the thermal and geometrical properties of Table 1. The resulting law is representative of the typical short duration tests in the CT3 facility.

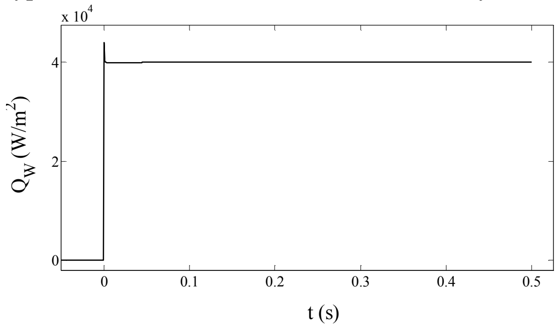


Fig. 4. Reconstructed wall heat flux with the 2D FEM model at y=H/2

The wall heat flux has been reconstructed in every time step along the left wall of the domain *(x=0)*, using the formulation of Eq. (14). Fig. 4 shows the $40000\;\; W/m^2$ wall heat flux evolution in the location *(0, H/2)*. The same result is obtained in the whole boundary, proving the efectiveness of the 2D FEM model.

WALL HEAT FLUX RECONSTRUCTION AROUND A DOUBLE-LAYERED AIRFOIL

The finite element model of the 2D unsteady heat conduction equation has been used to solve the double-layered cross sectional area of a rotor blade. The internal layer of the substrate is the metallic blade itself, and the external layer is the polyamide sheet over which the thin film is deposited (thickness $\approx 175\ \mu m$). The thermal properties of the two layers are those used in the validation test case (see Table 1). Fig. 5 shows the geometry under study, meshed for the solution with the Finite Element Method. Four reference points are defined in the leading edge, the trailing edge, the pressure side and the suction side.

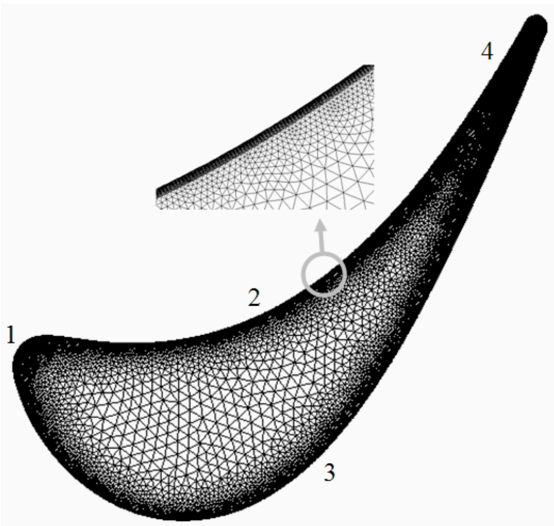


Fig. 5. FEM mesh of a double-layered rotor blade

The initial and boundary conditions imposed in this problem are:

$$T(\Omega,0) = 0^o C$$
$$T(\partial\Omega,t) = T_W(t), \qquad 0 < t(s) \le 0.5$$

where $T_W(t)$ is the same temperature evolution prescribed in the numerical validation test (see Fig. 3), linked to the reconstruction of a $40000\ W/m^2$ wall heat flux step in a 1D double-layered, semi-infinite substrate.

The solution of the 2D unsteady heat conduction equation allows computing the wall heat flux evolution in the whole boundary of the airfoil. The expected heat flux step is reconstructed in wide regions of the pressure side and suction side during the whole test duration: it proves that these regions fulfil the 1D hypothesis (see locations 2 and 3 in Fig. 6). However, the wall heat flux evolution around the leading edge and trailing edge does not behave as a step function: the initial $40000\ W/m^2$ heat flux reconstruction decays short after the commencement of the test (see locations 1 and 4 in Fig. 6). This is due to the presence of lateral heat conduction phenomena, mainly in the leading edge, and the failure of the semi-infinite hypothesis, specially in the trailing edge.

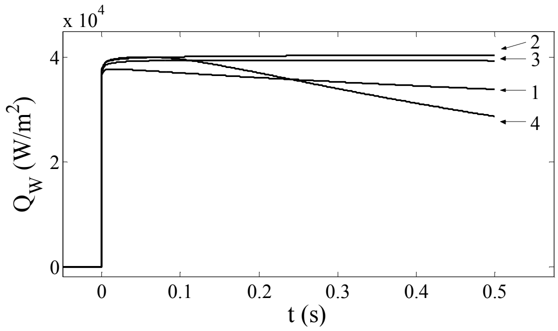


Fig. 6. Wall heat flux reconstruction at four locations

The wall heat flux computed at the end of the test (t_f=0.5 s) is represented around the airfoil boundary in Fig. 7.

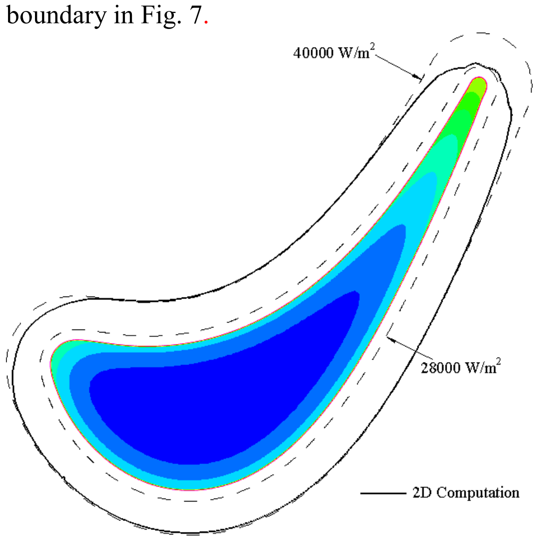


Fig. 7. Wall heat flux reconstruction at t_f=0.5 s

Fig. 7 shows very graphically that the leading and trailing edges are affected by 2D effects. The global approach that consists of solving the cross-sectional area of the airfoil allows computing correctly the wall heat flux in all the external boundary, regardless the existence of 1D or 2D local effects.

COMPUTATION OF THE INITIAL TEMPERATURE DISTRIBUTION OF A SHORT DURATION TEST

Heat transfer experiments carried out with thin film gauges consist of short duration tests, where a hot gas is suddenly released across the turbine test rig. Before this expansion occurs, the facility is accelerated to its nominal rotational speed (6500 rpm) in a process called run-up, that takes around 10 minutes. During this process, the blades, initially at thermal equilibrium with the ambient, heat up non uniformly. Fig. 8 shows the local increase of wall temperature, idealized by parabolic laws. The initial and final temperatures at each measurement location fit fairly well the experimental evidence.

Fig. 9 shows the typical temperature distribution around a rotor blade at CT3 facility, after the run-up process (t_f=600s, immediately before the blowdown test). A non-uniform temperature distribution is expected after the run up in the cross-sectional area of an airfoil. Nevertheless, the common 1D post-processing approach is not able to provide this initial temperature distribution. Usually, a uniform temperature distribution in the 1D domain, equal to the wall temperature before the blowdown at each measurement location, is considered.

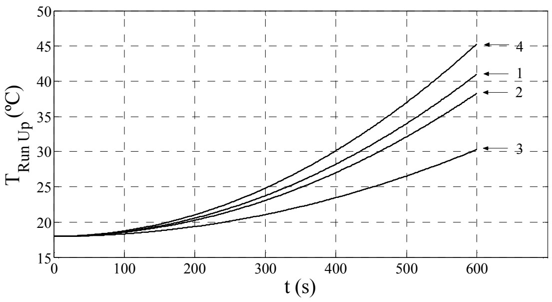


Fig. 8. Wall temperature evolution at four locations during the run-up process

The goal of this numerical test is to compute the two-dimensional temperature distribution in the cross sectional area of a rotor blade at the end of the run-up process. This solution must be employed as initial condition in the 2D post-processing approach of short-duration heat transfer experiments.

The 2D unsteady heat conduction equation has been solved in the same domain than the previous test case (see Fig. 5). The thermal properties of the two-layers are represented in Table 1. The initial and boundary conditions imposed in this problem are:

$$T(\Omega,0) = 18^o C$$
$$T(x,y,t) = T_{Run\ Up}(x,y,t) \qquad x,y \in \partial\Omega, \quad 0 < t(s) \le 600$$

The temperature distribution in the cross sectional area of the airfoil is shown in

Fig. 10. As expected, there exists a two dimensional gradient across the domain.

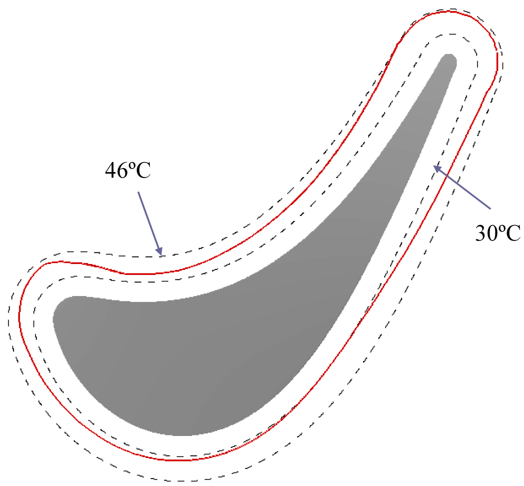


Fig. 9. Wall temperature distribution at the end of the run-up

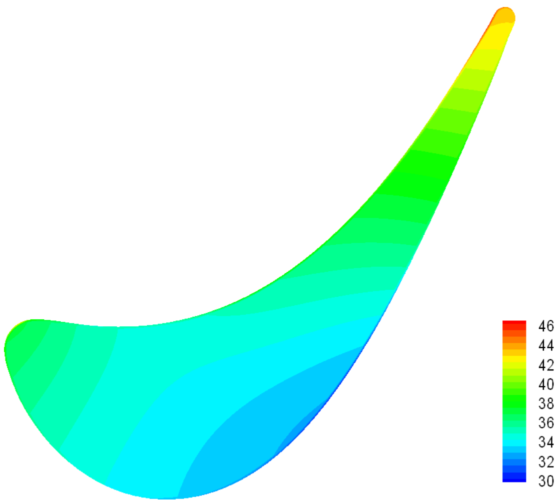
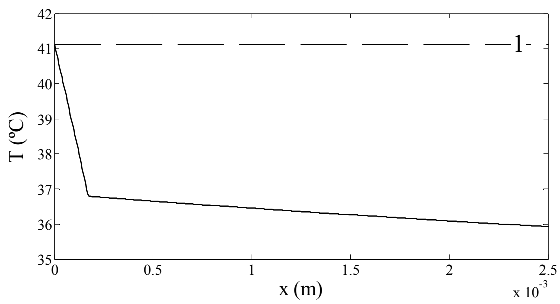


Fig. 10. Temperature distribution in the cross-sectional area of the airfoil at the end of the run-up

The temperature profile inside the blade, normal to the wall, has been extracted in the four reference points depicted in Fig. 11. The uniform initial temperature distribution commonly employed in the 1D approach has been represented as well. Huge initial temperature gradients exist before the heat transfer test. The influence of this gradients on the heat flux computation must be addressed in future works.



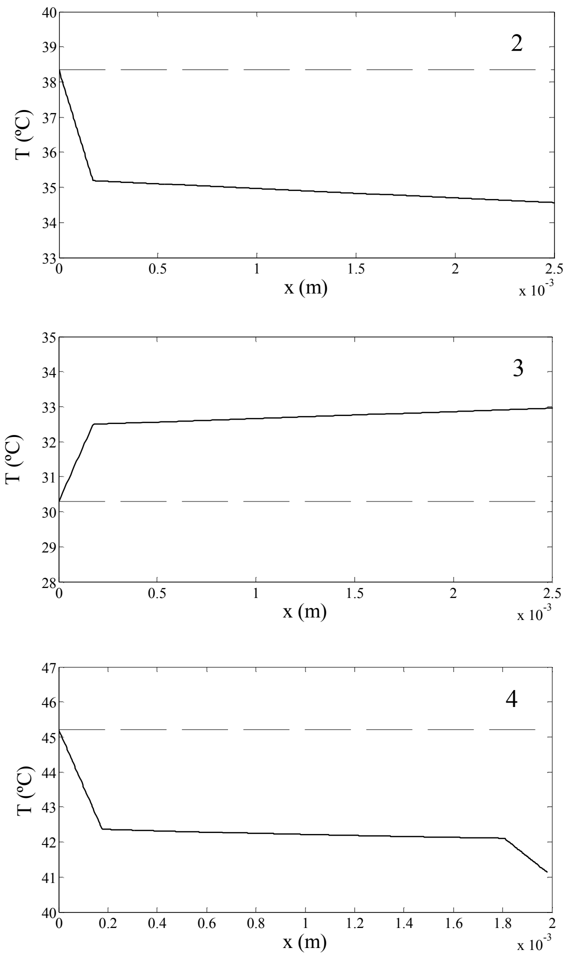


Fig. 11. 1D temperature profiles inside the blade at four locations

CONCLUSIONS

A novel methodology to post-process heat transfer experiments with double-layered thin film gauges has been proposed. It consists of implementing in the double-layered cross-sectional area of a turbine airfoil a finite element model of the two-dimensional unsteady heat conduction equation. Under analytical boundary conditions for 1D models, it has been demonstrated that the leading edge and the trailing edge of the airfoil do not fulfil the requirements for the 1D post-processing: lateral heat conduction phenomena appear, as a result of the curvature of the geometry and the absence of a semi-infinite substrate.

The effect of non uniform heating during a run-up process has been simulated, to compute the initial temperature distribution for short duration heat transfer experiments. Huge temperature gradients in the wall of the instrumented airfoil exist before the blowdown. Its influence on the heat flux computation must be studied.

The potential of the 2D post processing technique has been shown. The application of this data reduction approach to experimental data, and its comparison with one-layered results, is necessary to confirm the benefits of this technique.

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