DATA REDUCTION METHOD FOR THREE-DIMENSIONAL TRANSONIC FLOW MEASUREMENTS

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ABSTRACT

This paper describes evaluation of representative aerodynamic parameters from the results of three-dimensional transonic flow measurements. The evaluation method is based on a consistent balance of mass, momentum and energy.

INTRODUCTION

The data reduction method was developed to investigate plane cascades[1]. The results of traversing probe measurements downstream of the cascade are entrance data and two-dimensional flow is taken into account. The 5-equation system is derived (one equation for mass flow rate balance, two equations for momentum flow rate balance, one equation for energy flow balance, and one state equation of an ideal gas). This system was solved by a well-known procedure[1], in which a biquadratic equation for non-dimensional velocity M_* is derived. The proposed data reduction method was extended in [2] for total temperature distribution measurements. Further development of the data reduction method was carried out in [3], where representative aerodynamic parameters for the case of another gas injection are solved.

This paper aims to present the data reduction method when three-dimensional flow is taken into account.

THREE-DIMENSIONAL FLOW MEASUREMENTS

The set of data from three-dimensional traversing probe measurements (Fig.1) consists of distributions of

- static pressure, p_{yz}
- total pressure, p_{0yz}
- angle of flow between direction x and the velocity vector component projected to the x-y plane, γ_{yz}
- angle of flow between the velocity vector and velocity vector component projected to the x-y plane, α_{yz}.

Adiabatic flow is assumed, T_{0yz} = const. in the whole flow field.

DATA REDUCTION METHOD

The proposed data reduction method is based on consistent balances of mass, momentum and energy rate of ideal gas flow in the plane y-z. Reference parameters static pressure p, total pressure p_0, flow direction β, flow direction γ and non-dimensional velocity M_* are solved in plane y-z in the region A from y = 0 to y = t and from z = 0 to z = h.

By comparing with reference parameters in the other plane upstream of plane y-z it is possible to solve the kinetic energy loss coefficient, entropy increase, etc.

An important aspect of the data reduction method is the solution of integrals from data from experimental aerodynamic measurements. The integrals are :

mean mass flux

$$I_M = \frac{1}{A}\iint_A \rho_{yz}\left(w_x\right)_{yz} dy\, dz \qquad (1)$$

mean momentum flux normal to the traverse plane

$$I_A = \frac{1}{A}\iint_A \left[\rho_{yz}\left(w_x\right)_{yz}^2 + p_{yz}\right] dy\, dz \qquad (2)$$

mean momentum flux parallel to the traverse plane in direction y

$$I_C = \frac{1}{A}\iint_A \rho_{yx}\left(w_x\right)_{yz}\left(w_y\right)_{yz} dy\, dz \qquad (3)$$

and mean momentum flux parallel to the traverse plane in direction z

$$I_R = \frac{1}{A}\iint_A \rho_{yx}\left(w_x\right)_{yz}\left(w_z\right)_{yz} dy\, dz \qquad (4)$$

where $(w_x)_{yz}$, $(w_x)_{yz}$ and $(w_x)_{yz}$ are components of local velocity vector **w** (see Fig.2).

The set of 6 equations (1 mass flow rate balance, 3 momentum flow rate balance, 1 energy flow balance, 1 state equation of an ideal gas) forms the basis of the proposed data reduction method :

$$\rho w_x = I_M \tag{5}$$

$$\rho w_x^2 + p = I_A \tag{6}$$

$$\rho w_x w_y = I_C \tag{7}$$

$$\rho w_x w_z = I_R \tag{8}$$

$$w_x^2 + w_y^2 + w_z^2 = 2c_p(T_0 - T) \tag{9}$$

$$p = \rho r T \tag{10}$$

where all parameters (except constant specific heat capacity at constant pressure c_p and specific gas constant r and integrals I_M, I_A, I_C and I_R) are representative aerodynamic data. The set of equations from (5) to (10) contains 6 unknowns, and can be solved. After substitutions, the set can be reduced to a system of 5 equations which can be solved by a well-known procedure [1] and analysis from [4] and [5] can be applied.

After some algebra containing substitution

$$z = I_A - p = \frac{I_M^2}{\rho} \tag{11}$$

the quadratic equation is derived

$$\left(1 - \frac{r}{2c_p}\right) z^2 - I_A z + $$

$$+ T_0 r I_M^2 - \frac{r}{2c_p}\left(I_C^2 + I_R^2\right) = 0 \tag{12}$$

Eq.(12) has the following solutions

$$z\big|_{1,2} = \frac{I_A \pm \sqrt{D}}{2\left(1 - \frac{r}{2c_p}\right)} \tag{13}$$

where the discriminant is

$$D = I_A^2 -$$

$$- 4\left(1 - \frac{r}{2c_p}\right)\left[T_0 r I_M^2 - \frac{r}{2c_p}\left(I_C^2 + I_R^2\right)\right] \tag{14}$$

and where - (minus) holds for all subsonic solutions and transonic and supersonic solutions under the limit load regime, and + (plus) should be accepted in the case of a transonic and supersonic solution exceeding the limit load regime.

The representative parameters are solved static pressure

$$p\big|_{1,2} = I_A - z\big|_{1,2} \tag{15}$$

static density

$$\rho\big|_{1,2} = \frac{I_M^2}{z\big|_{1,2}} \tag{16}$$

static temperature

$$T\big|_{1,2} = \frac{p\big|_{1,2}}{\rho\big|_{1,2} r} \tag{17}$$

velocity components

$$w_x\big|_{1,2} = \frac{I_M}{\rho\big|_{1,2}} \tag{18}$$

$$w_y\big|_{1,2} = \frac{I_C}{I_M} \tag{19}$$

$$w_z\big|_{1,2} = \frac{I_R}{I_M} \tag{20}$$

velocity vector value (Fig.2)

$$w\big|_{1,2} = \sqrt{2c_p\left(T_0 - T\big|_{1,2}\right)} \tag{21}$$

arguments of velocity vector (Fig.2)

$$\alpha\big|_{1,2} = \arcsin\frac{w_z\big|_{1,2}}{w\big|_{1,2}} \tag{22}$$

$$\gamma\big|_{1,2} = \mathrm{arctg}\frac{w_y\big|_{1,2}}{w_x\big|_{1,2}} \tag{23}$$

Mach number

$$M\big|_{1,2} = \frac{w\big|_{1,2}}{\sqrt{\kappa r T\big|_{1,2}}} \tag{24}$$

total pressure

$$p_0\big|_{1,2} = p\big|_{1,2}\left[1 + \frac{\kappa - 1}{2}\left(M\big|_{1,2}\right)^2\right]^{\frac{\kappa}{\kappa - 1}} \tag{25}$$

When the total pressure $p_{0\,ref}$ in the reference plane upstream of the traversing plane is given, the isentropic Mach number is solved

$$M_{is}\big|_{1,2} = \sqrt{\frac{2}{\kappa - 1}\left[\left(\frac{p\big|_{1,2}}{p_{0ref}}\right)^{-\frac{\kappa - 1}{\kappa}} - 1\right]} \tag{26}$$

And the isentropic total temperature is

$$T_{is}\big|_{1,2} = T_0\left(\frac{p\big|_{1,2}}{p_{0ref}}\right)^{\frac{1-\kappa}{\kappa}} \tag{27}$$

loss coefficient

$$\zeta\big|_{1,2} = 1 - \frac{T_0 - T\big|_{1,2}}{T_0 - T_{is}\big|_{1,2}} \tag{28}$$

and the entropy increase is

$$s\big|_{1,2} - s_{ref} = r \ln\frac{p_{0ref}}{p_0\big|_{1,2}} \ . \tag{29}$$

DISCUSSION AND CONCLUSIONS

The data reduction method in principle simplifies the results of measurements to obtain representative aerodynamic parameters. The procedure has to be based on physical fundamentals - on balance laws. The reduction process leads to loss of detailed information. Further practical requirements concerning the solution of important quantities should therefore be formulated. The data reduction method is utilized in aerodynamic laboratories, where experience with the method has been obtained [6], [7]. New demands on numerical modelling, namely for evaluating the kinetic energy loss coefficient or the aerodynamic drag coefficient, open possibilities for application of the data reduction method in CFD.

The proposed data reduction method concerns 3D measurements in the traversing area and utilizes knowledge from previous developments of 2D traversing techniques. This approach makes it possible to determine which of two solutions (Eq.(13)) has to be taken into account for further evaluation of the parameters (Eqs. from (15) to (29)). In [4] it was proved that the sign + (plus) in Eq.(13) has to be for the case when the limit load regime is exceeded. The extensions for total temperature distribution [2] and for the concentration distribution of another gas [3] can certainly be incorporated into the proposed data reduction method.

The proposed data reduction method was verified for a model data set, and will be applied for measurements of the radial cascade and the rotating annular cascade. Further development of the data reduction method is possible in comparison with the approach by [8]. Development of analysis of uncertainities should be useful for applications of the traversing technique. Entropy analysis of the data reduction method will also be a contribution to this important aspect of evaluation techniques in high-speed aerodynamics.

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REFERENCES

[1] J. Amecke: Data Reduction of Wake Flow Measurements with Plane Cascades, AVA-Report 76 A 49, Göttingen, 1967 (in German)

[2] M.L.G. Oldfield, D.L. Schultz, J.H. Nicholson: Loss Measurements Using a Fast Traverse in ILPT Transient Cascade, Paper No.17, pp.0-25
In : Proceedings of 6th Meeting on Measuring Techniques in Transonic and Supersonic Flows in Cascades and Turbomachines, Lyon, 1981

[3] J. Amecke, P. Šafařík : Data Reduction of Wake Flow Measurements with Injection of an Other Gas, Paper No.3, pp.0-8
In: 12th Symposium on Measuring Techniques for Transonic and Supersonic Flow in Cascades and Turbomachines, Proceedings, Prague, 1994

[4] J. Amecke, P. Šafařík : Data Reduction Method of Wake Flow Measurements with Injection of an Other Gas, DLR-Forschungsbericht No.95-32, Göttingen, 1995

[5] P. Šafařík : Multicomponent Mixture Flows in Machine Flow Parts, Thesis, Prague, 1999 (in Czech)

[6] K. Freunreich, M. Jöcker, W. Höhn, T.H. Fransson, H.J. Rehder : Aerodynamic Performance of Two Isolated Turbine Stators in Transonic Annular Cascade Flow, pp.3-19
In : Third European Conference TURBOMACHINERY – FLUID DYNAMICS AND THERMODYNAMICS, Vol.A, ImechE Conference Transactions, London, 1999

[7] H.J. Rehder, M. Tiedemann, Y.X. Zhong, Z.L. Jiang : Displacement Effects of Side-Wall Boundary Layers on Measurements in a Turbine Cascade, pp.272-280
In : Aerothermodynamics of Internal Flows III, Proceedings of the Third ISAIF, World Publishing Corporation, Beijing, 1996

[8] A.J. Main, M.L.G. Oldifield, G.D. Lock, T.V. Jones : Free Vortex Theory for Efficiency Calculations from Annular Cascade Data, ASME Paper No. 95-GT-186, New York, 1995

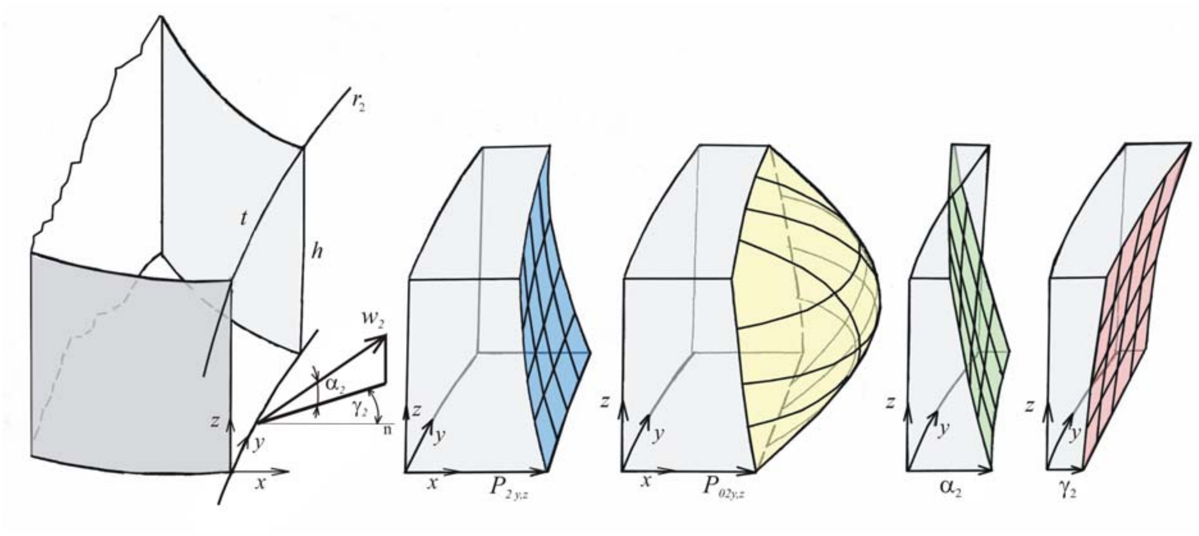

Fig.1. Distribution of flow field parameters.

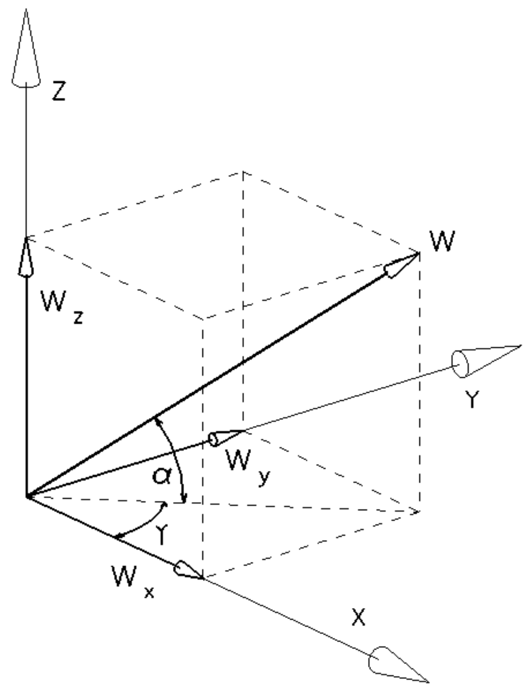

Fig.2. The coordinate system with components of velocity vector **w**.