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REYNOLDS STRESS MEASUREMENT WITH A SINGLE COMPONENT LASER DOPPLER ANEMOMETER

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ABSTRACT

Measurements of the 2D Reynolds stress tensor of a steady or periodic flow may be made using a rotated 1D probe. The use of this technique in LDA measurements is presented showing that the LDA is in fact an ideal instrument for this technique due to its known cosine response with angle. The rotated 1D technique is compared to 2D LDA measurements and it is demonstrated that a 1D system can be used to make 2D Reynolds stress measurements at a fraction of the capital cost.

INTRODUCTION

The measurement of the full 2D Reynolds Stress tensor typically requires multi-axis anemometry systems. By simultaneously measuring multiple velocity components, it is possible to directly calculate the correlation between the components of velocity fluctuation and thus to calculate the Reynolds Stress tensor. However, if the flow is time invariant, or if it is phase locked, it is possible to combine a series of measurements made at different probe orientations and derive the time averaged or ensemble averaged Reynolds Stress tensor. This has been demonstrated by Fujita and Kovasznay [1], Kuroumaru et al.[2] and Kool et al. [3] who have used the technique in thermal anemometry.

The adaptation of the method of Fujita and Kovasznay [1] to Laser Doppler Anemometry (LDA) is detailed here. It is demonstrated that the directional response of LDA makes it most suitable to this form of measurement. Moreover, the range of possible measurements obtainable from a 1D system is greatly enhanced with significant financial savings on equipment purchase. The data collected at multiple probe angles can also be utilised to enhance the mean flow measurements.

NOMENCLATURE

- M measured quantity
- N number of probe angles
- S error
- U mean velocity magnitude
- i index
- m fluctuation of measured quantity
- u streamwise fluctuation
- v fluctuation normal to stream
- α angle of probe
- θ angle between probe and mean flow
- τ bar passing period
- $\overline{\ }$ time mean
- <> ensemble mean

DERIVATION

The velocity measured by LDA is the component of velocity in the plane of the intersecting beams and normal to the fringe pattern. If the plane of the beams is rotated relative to the instantaneous flow velocity vector (U) as shown in Figure 1, then only the instantaneous component of velocity in the plane of the intersecting beams (M) will be measured according to the cosine relationship

$$M = U\cos(\theta) \qquad (1)$$

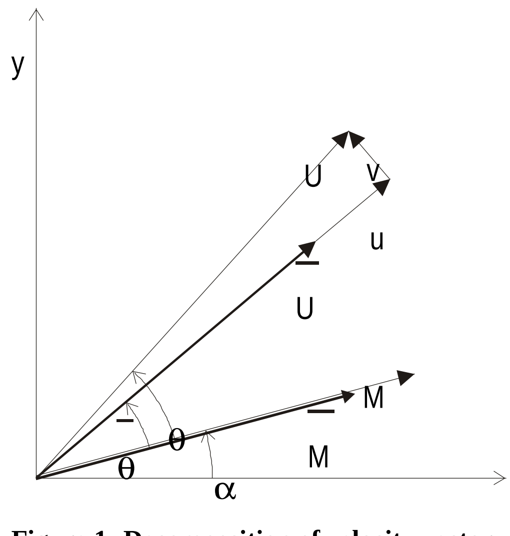

Figure 1: Decomposition of velocity vector.

The instantaneous velocity component measured by the LDA (M) can also be written in terms of the mean flow vector ($\overline{U}$), the instantaneous fluctuation components normal (v) and parallel (u) to the local mean flow and the angle between the probe and the mean flow ($\overline{\theta}$) according to

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$$M = (\overline{U} + u)\cos(\overline{\theta}) - v\sin(\overline{\theta}) \tag{2}$$

and the mean measured velocity, $\overline{M}$, may be written

$$\overline{M} = \overline{U}\cos(\overline{\theta}) \tag{3}$$

A relationship between the variance of the measured velocity, $\overline{m^2}$, and the Reynolds Stress tensor (aligned to the mean flow direction) may be obtained by subtracting (3) from (2), squaring the result and averaging to get

$$\begin{aligned}\overline{m^2} = \overline{u^2}\cos^2(\overline{\theta}) + \overline{v^2}\sin^2(\overline{\theta}) \\ -2\overline{uv}\sin(\overline{\theta})\cos(\overline{\theta})\end{aligned} \tag{4}$$

If data is acquired at three probe angles then the Reynolds stress tensor may be calculated directly from equation (4). If more than three probe angles are used, then for a given number of acquired data points, the quality of the measurement can be improved by a least squares fit to the data obtained by solving the following linear system

$$\sum_{i=1}^{N} A(\overline{\theta}_i)\cdot\begin{bmatrix}\overline{u^2}\\ \overline{v^2}\\ \overline{uv}\end{bmatrix} = \sum_{i=1}^{N}\overline{m_i^2}\begin{bmatrix}\cos^2(\overline{\theta}_i)\\ \sin^2(\overline{\theta}_i)\\ \cos(\overline{\theta}_i)\sin(\overline{\theta}_i)\end{bmatrix} \tag{5}$$

where

$$A(\overline{\theta}_i) = \begin{bmatrix}\cos^4(\overline{\theta}_i) & \cos^2(\overline{\theta}_i)\sin^2(\overline{\theta}_i) & \cos^3(\overline{\theta}_i)\sin(\overline{\theta}_i)\\ \cos^2(\overline{\theta}_i)\sin^2(\overline{\theta}_i) & \sin^4(\overline{\theta}_i) & \cos(\overline{\theta}_i)\sin^3(\overline{\theta}_i)\\ \cos^3(\overline{\theta}_i)\sin(\overline{\theta}_i) & \cos(\overline{\theta}_i)\sin^3(\overline{\theta}_i) & \cos^2(\overline{\theta}_i)\sin^2(\overline{\theta}_i)\end{bmatrix}$$

However, in practical measurements, the value of $\overline{\theta}$ is not known as only the probe angle (α_i) and measured data ($\overline{M}$ and $\overline{m^2}$) are known. Since the measurement of the turbulent quantities requires measurements at multiple probe angles, the mean flow may first be determined from this data and from this the value of $\overline{\theta}$ is determined.

The quality of the mean flow measurements can be improved as suggested by Dambach and Hodson [4]. If more than three probe orientations are used all the data may be used by minimising the error (S) between the mean velocities measured at the N probe angles and the functional relationship for the angular response of the LDA given by equation (1). The resulting function to be minimised in this case, is therefore

$$S = \sum_{i=1}^{N}\left(\overline{M}_i - \overline{U}\cos(\overline{\theta}_i)\right)^2 \tag{6}$$

This minimisation may be performed numerically as the function is non-linear in the variables of interest namely $\overline{U}$ and $\overline{\theta}$. Furthermore, with knowledge of the mean flow direction, it is possible to transform the Reynolds stresses, calculated in the direction of the mean velocity vector, to any co-ordinate system using a standard co-ordinate transformation.

The derivation presented above is for a stationary flow. However, for a periodic flow, the ensemble average 2D Reynolds stress tensor may be determined by the same method. The flow period is first divided into a series of time segments. The mean and variance is calculated for each of these time segments for each of the probe orientations. By using ensemble average data in the place of time average data in equations (5) and (6), the 2D Reynolds stress tensor and the mean flow vector at each of these time segments may then be found and the ensemble averaged quantities thus determined.

EVALUATION OF TECHNIQUE

In order to validate the experimental technique, the flow at the inlet to a turbine cascade downstream of a moving bar wake generator was measured using the rotated 1D LDA technique described above and using conventional 2D LDA.

The wake generator consisted of a series 2.05mm diameter stainless steel bars suspended between two belts. A variable speed DC motor was used to drive the belts through a system of pulleys so that the bars traversed the inlet to the cascade. The experimental facility is sketched in Figure 2 and further details may be found in Schulte and Hodson [5].

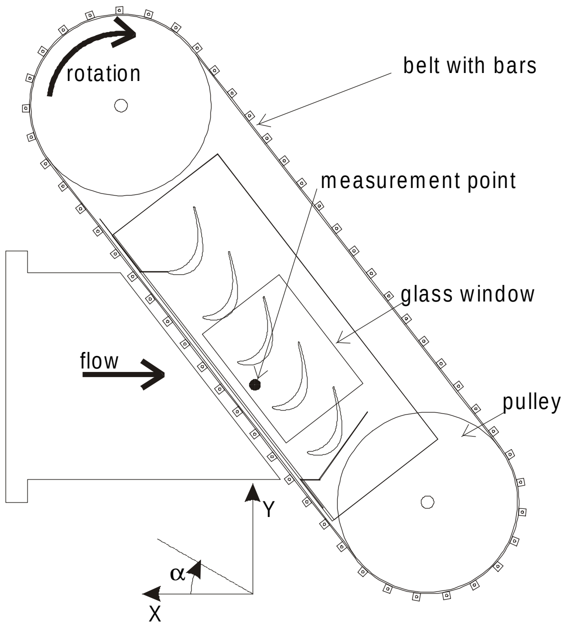

Figure 2: Sketch of moving bar wake generator.

The cascade and bars were set at an incidence of 37.7° to the inlet flow. The bars were spaced 158 diameters apart and 35 diameters axially upstream of the cascade. The bar passing frequency was set to 22.3Hz and the ratio of bar speed to inlet flow speed was set to 0.83. The measurement station was in the plane of the blade leading edges and at mid pitch of the cascade (see Figure 2). The

Reynolds number based on relative flow velocity and bar diameter was 1.7×10^3.

A 5W Argon-Ion laser was used in conjunction with a Dantec Fibre Flow unit, which contained a colour separator and brag cell. A 2D 85mm probe with 500mm focal length was used. The 514.5nm beam, which was used for the 1D measurements, had a measuring volume of 0.077mm diameter by 1.016mm long. The 488nm beam, used in conjunction with the 514.5nm beam for the 2D measurements had a measuring volume 0.0073mm diameter by 0.963mm long. A backscatter configuration was used. The photo-multiplier outputs were processed by Dantec BSA signal processors.

At each probe orientation, 120 000 samples were collected over a maximum of 5000 wake passing events. A trigger signal, generated at each bar passing, was used to time stamp the collected data. Ensemble averaging of the measured signal was then performed by dividing the wake passing period into 128 segments and calculating the mean and variance for each time segment. Only the 514.5nm beam was used for the rotated 1D measurements. For the 2D measurements, coincidence filtering was performed by software with a coincidence interval of 0.005ms and the Reynolds stress was calculated for each time bin.

The flow was seeded with smoke generated from mineral oil, which was injected into the wind tunnel through the trailing edge of a streamlined injector tube. The point of injection was approximately 3m upstream of the final screen and contraction of the wind tunnel, thus the effect on the flow was insignificant.

The ensemble-averaged results of the 1D rotated LDA and the 2D LDA are compared in Figure 3, with only the portion of the period in which the wake passes shown. The Reynolds stress tensor is aligned with the x-y axes of Figure 2. Excellent agreement is evident for the ensemble mean data of the two techniques. For the turbulent statistics, however, the agreement is seen to improve as the number of angles and thus the total number of samples increases. Estimating the error of the rotated 1D technique as the average variance from the 2D LDA measurement and defining the cost of the measurement as the total number of samples used, it is possible to plot the quality of the 1D measurements as a function of cost. Figure 4 shows the error in <uv> for a range of numbers of probe angles, N. Similar results were obtained for $<u^2>$ and $<v^2>$.The highest errors, at all costs, were obtained for case with N=3. This demonstrates the benefit of using the least squares approach to improve the quality of the measurements. However, the case of highest N does not give the lowest error. This indicates that there is an optimum N for a given cost. For a constant cost experiment, increasing N will reduce the number of samples in each bin from which statistics are calculated. With inaccurate statistics, the quality of the least squares fit is compromised.

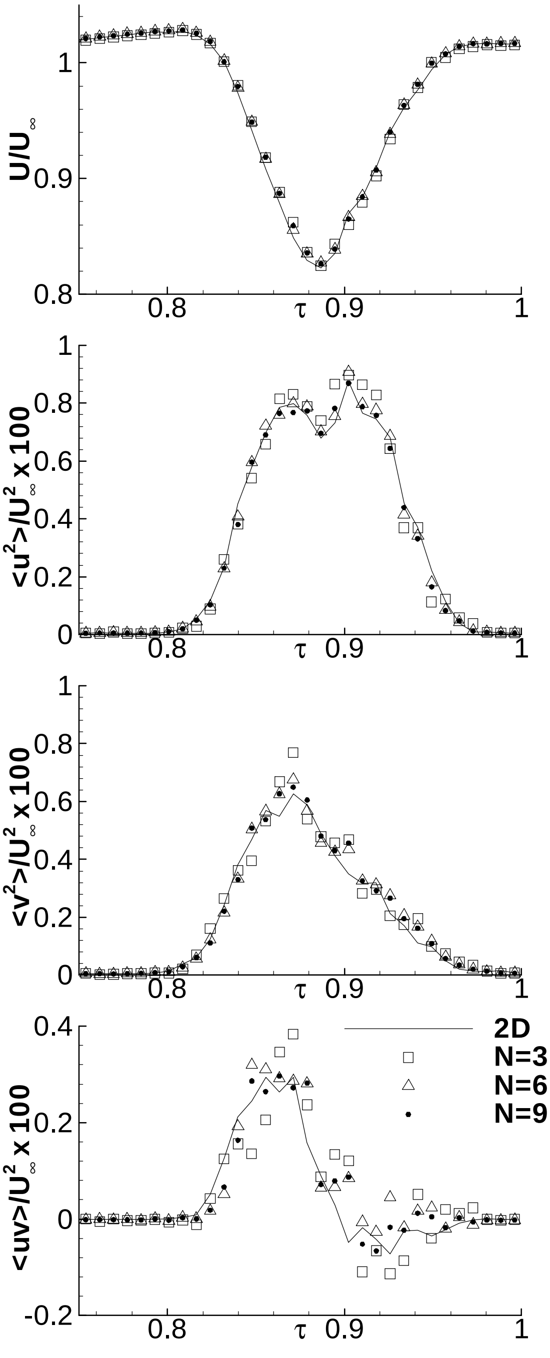

Figure 3:Comparison of conventional 2D LDA measurements and rotated 1D LDA for 3, 6 and 12 probe angles. The measurements are ensemble-averaged data taken at one measurement location.

DISCUSSION

Although the form of equation (4) matches that presented by Fujita and Kovasznay [1], it is more precise because the angular response of the LDA is known. The derivation presented by Fujita and Kovasznay [1] employed a calibrated functional fit for the angular response of the HWA therefore, their derivation required a Taylor expansion and a small angle approximation. This requires the fluctuation to be small relative to the mean. However, when the functional response is known exactly, there is no such limitation and the technique is applicable to flows with large fluctuations relative to the mean. It should be noted that the least squares formula presented by Fujita and Kovasznay [1] assumes that the probe angles are symmetrically distributed about the flow. The least squares fit presented in equation (5) is a more general case and places no restriction on the selection of probe angles.

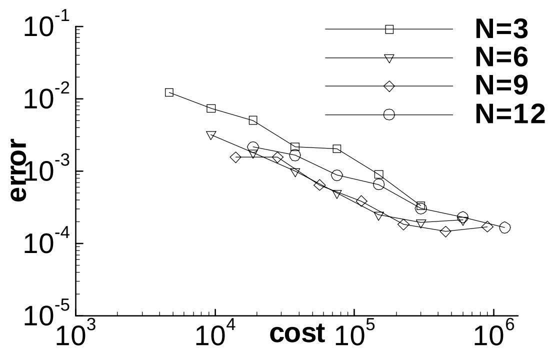

Figure 4:Error in <uv> for rotated 1D LDA for different numbers of probe angles. The 2D LDA measurement is taken as the datum

This technique retains significant advantages over rotated hot-wires as discussed by Fujita and Kovasznay [1]. The spatial resolution of the rotated 1D LDA is far superior to that of a rotated single hot wire and indeed a X-wire. The measuring volume of the rotated LDA is the same as that for a single orientation and has a diameter of 0.077mm whereas a rotated single wire sweeps out a circle with diameter equal to the wire length normal to the flow direction where a typical hot wire is 1mm long. This means that the rotated hot wire cannot be used to make boundary layer measurements except in very large scale experiments, which are impractical for turbomachinery research. Furthermore, unlike the hot wire technique, there is no directional ambiguity in the LDA measurements and this remains true for the technique described here.

CONCLUSIONS

A technique for using a single component LDA system to measure the 2D Reynolds Stress tensor has been presented and found to be in excellent agreement with the conventional 2D LDA measurements. This technique allows the functionality of 2D LDA at a capital cost comparable to a single component system.

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