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DISCHARGE COEFFICIENTS OF NOZZLE GUIDE VANE FILM COOLING HOLES WITH ENGINE-REPRESENTATIVE EXTERNAL FLOW

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Abstract

This paper discusses the measurement of the discharge coefficients of film cooling holes in a gas turbine nozzle guide vane (NGV) with external flow. The measurements are made in a large annular blowdown cascade capable of producing engine-representative external Reynolds and Mach numbers. To provide a datum for studying the influence of external flow, the hole discharge coefficients are also measured with zero external flow.

A set of accurately calibrated orifice plates are used to provide a known, controllable mass flow of coolant. Flow is required over a pressure range encompassing the engine design value, to any number of open film cooling holes from one to the full configuration (more than three hundred). At low pressure ratios - such as across a 'shower-head' leading edge cooling configuration - considerable care is required to ensure accurate results. Furthermore, an effective method of sealing unused holes, without causing aerodynamic interference, is necessary. These aspects of the experimental work will be discussed in the paper. Initial results show that discharge coefficients can be increased by some external flow conditions but decreased by others.

1 Nomenclature

1.1 Symbols

A	area
a	local speed of sound
C_d	discharge coefficient
d	diameter
l	hole length
$\dot{m}$	mass flow rate
M	Mach number
P	pressure
PR	pressure ratio across hole $= \dfrac{P_{oc}}{P_m}$
R	gas constant
Re	Reynolds number
T	temperature
u	velocity
α	angle of orientation
γ	ratio of specific heats
ρ	density
θ	angle of inclination

1.2 Subscripts

c	coolant
ca	air coolant
cf	foreign gas coolant
m	mainstream
o	total

2 Introduction

Blade film cooling permits an increase in turbine entry temperature by reducing the freestream-to-blade heat transfer, which, in turn, results in a higher power output and increased turbine efficiency. This is offset by decreased output due to the coolant bypassing certain engine stages - thus passing through the remaining turbines at a lower temperature than it otherwise would do - and losing total pressure as it is ducted to the blade. Furthermore, the process of mixing the coolant with the mainstream air after ejection leads to a reduction in aerodynamic efficiency. Consequently, successful implementation of a cooling system involves careful planning in order to achieve effective cooling (leading to maximum turbine entry temperature) without leading to a dramatic performance penalty.

A film cooling system is designed to supply the mass flow rate of coolant needed to provide the desired cooling performance. Exceeding this value means excessive usage of coolant and hence an unnecessary decrease in engine efficiency, whilst falling short of it could lead to local hot spots and, consequently, a shorter blade life. For a first stage nozzle guide vane, the coolant-to-mainstream pressure ratio is usually set by the compressor exit pressure, so the design problem hinges on correct hole sizing in order to provide the requisite coolant flow. This requires reliable discharge coefficient, C_d, data covering the particular geometry and flow.

The discharge coefficient is by no means invariant, depending on both the local geometry and the flow conditions upstream and downstream of the hole (Figure 1). It should be noted that hole geometries are chosen to maximise cooling effectiveness, with inclined holes often being used as higher effectiveness levels are achieved for shallow injection angles than for normal injection (Foster and Lampard, 1980). Furthermore, 'flared' or 'fanned' holes are often used in film cooling arrays as they reduce the momentum of the jet at exit, thereby improving the film cooling performance further (Hay and Lampard, 1995).

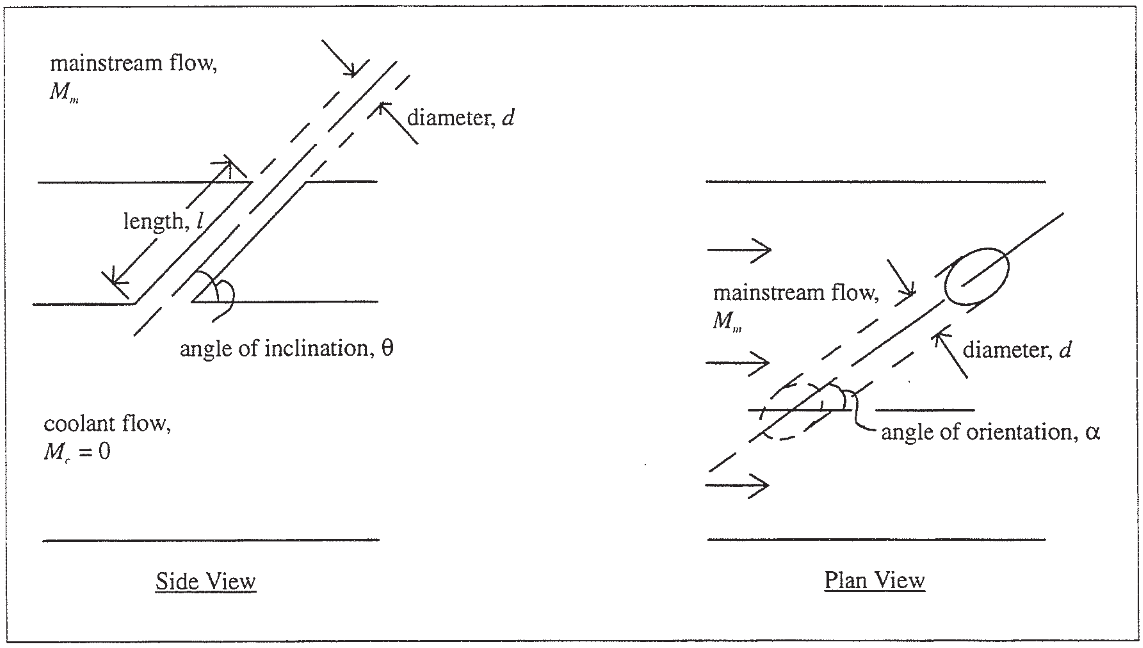

Figure 1 : Cooling Hole Geometric and Flow Parameters

2.1 Measuring Discharge Coefficients

The C_d of a cooling hole can be obtained by either treating the inlet and outlet flows separately and then using a simple 'loss coefficient' type analysis (that includes mixing within the hole), by modifying a basic hole C_d for the effects of geometric variations and the presence of crossflows, or by directly measuring the C_d of a hole having the correct geometry under the design flow conditions. The success of the first method depends on the lack of significant interactions between the various elements, which generally limits its application to simple cases; the analysis of a plenum-fed, normal cooling hole by Andrews and Mkpadi (1983) is a good example for this case. The second technique has been found to accurately model individual effects - as demonstrated by Tillman and Jen (1984), Hay, Khaldi and Lampard (1987), McGreehan and Schotsch (1988) and Hay, Lampard and Khaldi (1994) - although it is, not surprisingly, less successful under more complex conditions. The final approach is, obviously, ideal for obtaining coolant hole discharge coefficients under design conditions, although it can be difficult to achieve this accurately. The work described here uses this approach.

3 Geometrical and Flow Parameters

The NGV row being studied in the CHTT is representative of a first stage high pressure section of a modern aero-engine. The cooling geometry being studied consists of fourteen rows of holes, fed from one of two internal cavities. These cavities are independently supplied with coolant gas so as to maintain a coolant-to-mainstream total pressure ratio, P_{oc}/P_{om}, of 1.02. The 'forward' cavity supplies twelve of the fourteen rows - a total of almost three hundred coolant holes - positioned around the leading edge and the early regions of the pressure and suction surfaces. The 'rear' cavity feeds the two rows nearest the trailing edge on the pressure surface, a total of over fifty coolant holes.

It is worth noting that the range of flow conditions that are present amongst the different rows and holes is very diverse. For example, the mainstream (i.e. external) Mach number varies between 0.04 and 1.1 around the blade surface, whilst the pressure ratio across the hole, $PR = P_{oc}/P_m$, is anywhere from 1.02 to 1.77. The hole geometries are equally variant, with the angles of orientation, α, and inclination, θ, of the holes (see Figure 1) varying in the ranges $0^\circ < \alpha < 80^\circ$ and $10^\circ < \theta < 90^\circ$.

Furthermore, although the data presented in this paper is for cylindrical cooling holes, the test blades can be readily modified to model other geometries.

4 Test Facility

The experimental work summarised in this report was carried out in the Cold Heat Transfer Tunnel (CHTT) (Martinez-Botas , Main, Lock and Jones, 1993) in Oxford. The facility is an annular cascade of 36 NGVs at 1.4 times engine size, resulting in good spatial resolution on all measurements taken. Table 1 gives the salient CHTT NGV dimensions.

Mid-span axial chord	0.0664 m
Mean pitch at exit	0.09718 m
Span at exit	0.08076 m
Turning angle	73°
Throat area	0.08056 m^2
Mean blade diameter	1.113 m

Table 1 : CHTT NGV Details

The CHTT is a short duration, transonic test facility, capable of producing engine representative Reynolds and Mach numbers, but also, being an annular cascade of NGV blades, models the three-dimensional flow patterns found in modern aero-engines, including all secondary flow phenomena.

Moreover, the coolant system design allows the engine density ratio, ρ_c/ρ_m, and blowing parameter, $\rho_c u_c/\rho_m u_m$, to be matched to engine values and varied about these values.

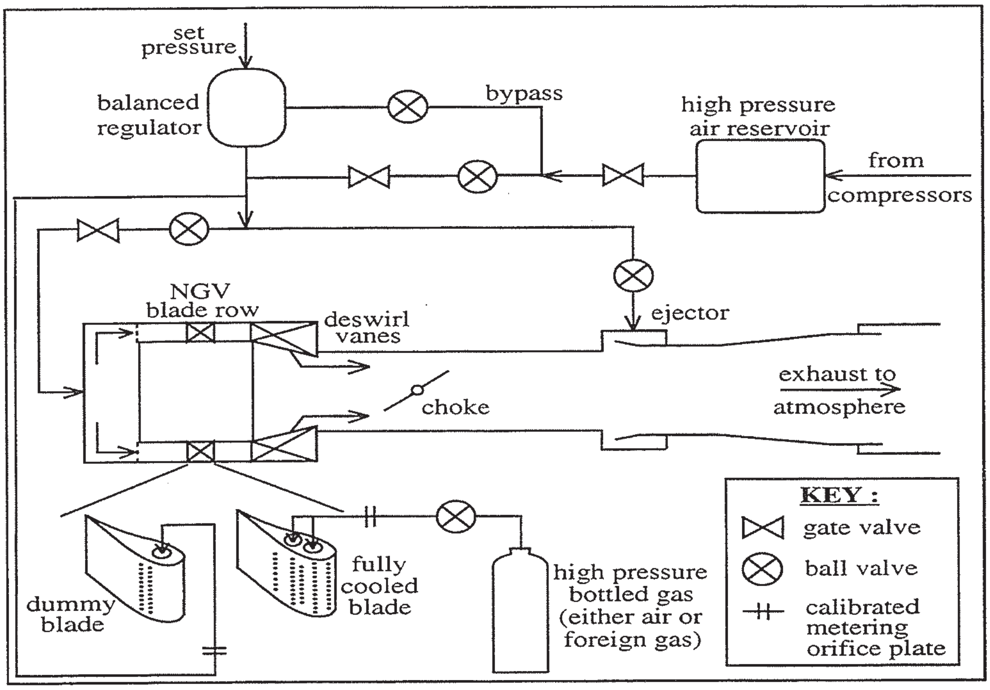

Figure 2 : Schematic Diagram of the CHTT

As illustrated in the schematic diagram of the CHTT in Figure 2, air flows via a balanced regulator - which ensures a constant total pressure upstream of the cascade - from the high pressure reservoir to an upstream plenum. Prior to operation the high pressure air reservoir typically sits at a pressure of about 2.7 MPa, falling to 2.0 MPa during 5 seconds of run time. It is the capacity of this reservoir that limits the maximum operation time. The flow then passes through a perforated distribution plate which ensures circumferential uniformity of the flow, whilst also creating a high level of freestream turbulence (~13% at NGV inlet), thereby simulating the exit conditions of a typical aero-engine combustor. The air is then accelerated through the cascade, gaining a swirl velocity in the process. The swirl is subsequently removed in the deswirl section before the air exhausts to atmosphere. A downstream choke, in combination with an ejector system, allows the independent variation of Reynolds and Mach numbers, or, equivalently, the upstream and downstream pressures can be independently and continuously varied.

All 36 blades in the annulus have the same surface geometry. The test blade and the two adjacent blades have the full film cooling configuration (Figure 3), whilst the remaining 33 NGVs have only 4 rows of cooling holes, fed from a single internal 'cavity'. Those blades having the simplified cooling geometry - 'dummy' blades - are present to provide periodicity of flow around the cascade, by matching aerodynamic blockage, whilst providing a substantial saving in manufacturing costs.

The coolant (either air or foreign gas, the latter of which is a dense gas used to simulate the lower coolant temperature; see Section 5) is supplied to blades having the full cooling geometry from high pressure bottles (Figure 2), with the flow being regulated by use of choked calibrated metering orifice plates. Having passed through the orifice plate, the coolant flows into the internal cavity of the blade where it stagnates before flowing out through the (open) film cooling holes and mixes with the mainstream flow. The coolant-to-mainstream pressure ratio can be varied, whilst the use of air or foreign gas as coolant permits data to be collected for two different coolant-to-mainstream density ratios.

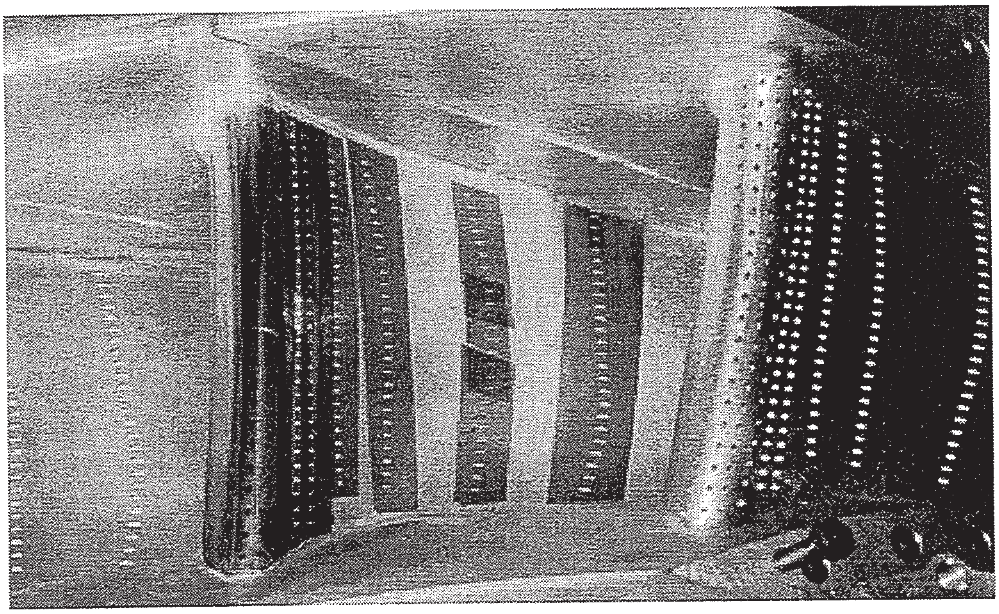

Figure 3 : CHTT NGV having the Full Film Cooling Geometry. The Centre Blade has all but Three Holes Sealed with Kapton Tape

5 Theory

5.1 Flow Measurement

If it is assumed that one has an accurate method for the control and measurement of the actual mass flow rate to the film cooling holes, then only the ideal mass flow rate is required in order to determine the discharge coefficient of those holes. Furthermore, if the static pressure at the exit (i.e. downstream) from the film cooling holes, P_m, and the cavity (upstream) total pressure and temperature, P_{oc} and T_{oc}, are known, then the isentropic flow equations can be used to determine the ideal mass flow rate and, hence, the C_d. This is summarised in the following equations :

$$\dot{m} = A.\rho_c .u_c$$

$$\dot{m}_{ideal} = A.\frac{\rho_c}{\rho_{oc}}.M\sqrt{\frac{T_c}{T_{oc}}}.\rho_{oc}\sqrt{\gamma R T_{oc}}$$

$$= A.\left(\frac{P_c}{P_{oc}}\right)^{\frac{1}{\gamma}}.\sqrt{\frac{2}{\gamma-1}\left[\left(\frac{P_c}{P_{oc}}\right)^{-\frac{\gamma-1}{\gamma}}-1\right]\left(\frac{P_c}{P_{oc}}\right)^{\frac{\gamma-1}{\gamma}}}.\rho_{oc}\sqrt{\gamma\frac{P_{oc}}{\rho_{oc}}}$$

$$= A.\left(\frac{P_m}{P_{oc}}\right)^{\frac{1}{\gamma}}.\frac{P_{oc}}{\sqrt{RT_{oc}}}.\sqrt{\frac{2\gamma}{\gamma-1}.\left[1-\left(\frac{P_m}{P_{oc}}\right)^{\frac{\gamma-1}{\gamma}}\right]},$$

since $P_c = P_m$ and $\rho_{oc} = P_{oc}/RT_{oc}$. It should be realised that if the hole or orifice becomes choked (i.e. $M = 1$) the above simplifies to the choked expression :

$$\dot{m}_{ideal} = \beta.A.P_{oc}.\sqrt{\frac{\gamma}{RT_{oc}}},$$

where $$\beta = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} = 0.578704 \text{ for } \gamma = 1.4.$$

The isentropic Mach number distribution over the blade surface is known from experiment (Martinez-Botas et al., 1993) and so the static pressure at any location on the surface (e.g. at the location of a film cooling hole) can be determined from the total pressure upstream of the cascade. Consequently, the ideal mass flow can be determined from the pressure upstream of the cascade, the Mach number distribution over the blade surface, and the pressure and temperature within the cavity. Therefore, by passing a known mass flow through an instrumented blade, whilst recording the flow parameters mentioned above and ensuring the flow is steady, a value for the appropriate discharge coefficient can be obtained. However, it should be noted that the total cavity pressures and temperatures are required and that the measured pressure and temperature may differ from the total condition if the internal velocity is significant.

5.2 Simulation of Engine Conditions

Due to the difficulty in reproducing the engine freestream-to-coolant temperature ratio in the CHTT, a 'foreign gas' coolant is employed in order to match the engine density ratio. It was demonstrated by Teekaram, Forth and Jones (1989) that simulation of this density ratio is possible using a foreign injection gas. For optimum aerodynamic simulation, the foreign gas should have a ratio of specific heats equal to that of air ($\gamma = 1.4$) but a higher density than air, at the same temperature, in order to simulate the cold cooling gas. The gas chosen for such a role is a mixture of sulphur hexafluoride (SF_6; 30.9% by weight) and argon (Ar; 60.1% by weight). Consequently, blades having the full cooling geometry are cooled with foreign gas when matching the engine freestream-to-coolant density ratio and air when investigating the effect of altering this density ratio. Unfortunately, the employment of a foreign injection gas makes it impossible to simultaneously match all aerodynamic parameters, and the most important must be chosen. In this instance, they are blockage at the cascade throat, total and static pressure ratios (and hence momentum flux ratios) in the mainstream and coolant flows, and local Mach numbers in the two flows. As the coolant mixes in a turbulent environment with the mainstream, Reynolds number is less important.

It should be realised that it is impossible to simultaneously match blockage and velocity ratios in all cases. Furthermore, during the course of this investigation it became apparent that the simultaneous matching of both the engine pressure ratios and the scaled engine mass flow rates could not be achieved. It was decided that the blockage and pressure ratios were the important parameters, and that these should, therefore, be matched if possible.

Matching blockage implies that the ratio of the coolant flow area to the mainstream flow area, A_c/A_m, at the mainstream throat is the same in all cases (i.e. the blockage is matched for different coolants). The static pressure at the throat and the ratio of specific heats are the same for both the coolant and the mainstream, so in order to match the ratio A_c/A_m, for a given mainstream flow, the product ($\dot{m}\sqrt{RT}/M$) should be the same for all coolants. However, matching $\dot{m}\sqrt{RT}$ and the Mach number, M, results in the momentum flux :

$$\dot{m}_c u_c = \dot{m}_c . M_c a_c = \dot{m}_c . M_c \sqrt{\gamma R_c T_c}$$

also being conserved. Consequently, the total and static pressures, and the total to static temperature ratios are the same for all coolants, thereby achieving similarity of all the key parameters. Furthermore, the conservation of the coolant-to-freestream momentum flux ratio not only provides the correct aerodynamic modeling but is also the necessary requirement for the correct modeling in heat transfer experiments (Teekaram et al., 1989).

Applying the above criteria to the experiments carried out in the CHTT, air and foreign gas cooling are correctly modelled if :

$$\dot{m}_{ca} = \dot{m}_{cf}\sqrt{\frac{R_{cf}T_{o,cf}}{R_{ca}T_{o,ca}}} \quad ,$$

which simplifies to

$$\dot{m}_{ca} = \dot{m}_{cf}\sqrt{\frac{R_{cf}}{R_{ca}}}$$

if the air and foreign gas coolants are at the same total temperature. The flow rate of coolant to the dummy blades, which are always air cooled, is also defined by the above relationship.

5.3 Experimental Technique

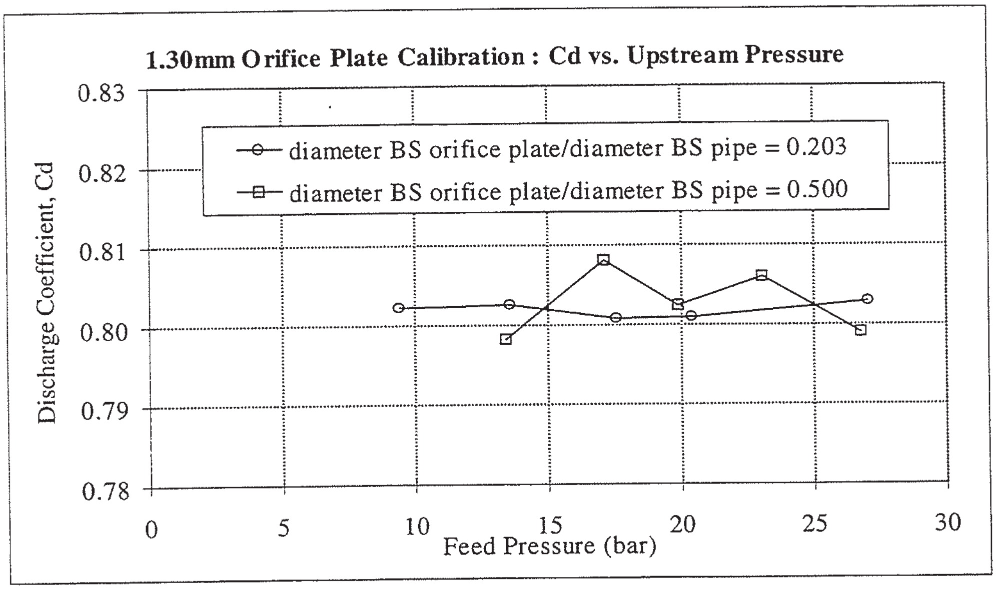

Figure 4 : Illustration of Achievable Orifice Plate Calibration Accuracy when Using a BS Calibration Rig

The two principal aspects to the establishment of a working experimental arrangement now become apparent. Firstly, a set of accurately calibrated orifice plates were required in order to provide known, controllable coolant mass flow rates. These were produced by calibrating orifices against two differently sized British Standard plates (with BS orifice-to-pipe diameter ratios of 0.203 and 0.500), in a BS calibration rig, to BS 1042 specifications (British Standards Institution, 1981). The double calibration provided evidence of the accuracy that is achieved using this method (Figure 4). Orifice sizes were chosen such that flow could be provided, over a pressure range encompassing the design value, to any number of open holes from one to an entire cavity. As such, mass flow rates from 0.050 to 25.0 g s^{-1} were required, which, due to limits on the range of supply pressure available, meant that calibrated orifice plates from 0.32 to 3.80 mm diameter had to be produced. Secondly, an effective method of selectively sealing-off holes had to be developed. This was achieved using kapton sheet and 3M VHB (Very High Bond) adhesive tape; this novel usage of the kapton allowed the application of relatively large cavity-to-external pressure ratios (up to ~3) without leakage, whilst minimising the aerodynamic interference (total 'tape' thickness of only 40μm) (Figure 3).

The engine representative coolant flow could then be readily achieved by using the foreign gas as coolant (to simulate the engine freestream-to-coolant density ratio) and the calibrated orifice plates (to regulate the coolant mass flow rate). Furthermore, the effect of varying the density ratio, ρ_c/ρ_m, and the blowing rate, $\rho_c u_c/\rho_m u_m$, could be investigated for any combination of open film cooling holes by varying the coolant used and the pressure ratio across the hole respectively. However, it should be noted that it was not possible to sweep through a range of blowing rates due to the long settling times required at low coolant flow rates, and so steady state tests were always performed. The pressure transducer and thermocouple outputs were sent via CAS 8A amplifiers to an A/D card within a computer. Analysis was carried out using the Mathworks Matlab and Microsoft Excel packages.

6 Results and Conclusions

6.1 Pressure Measurement Accuracies

To provide a datum for studying the influence of external flow, the hole discharge coefficients were also measured with zero external flow. Figure 5 illustrates the good agreement in the experimentally determined discharge coefficients of different combinations of open film cooling holes within a row when using different metering orifices, which supports the earlier claims on the attainable accuracy when calibrating choked orifice plates (Figure 4).

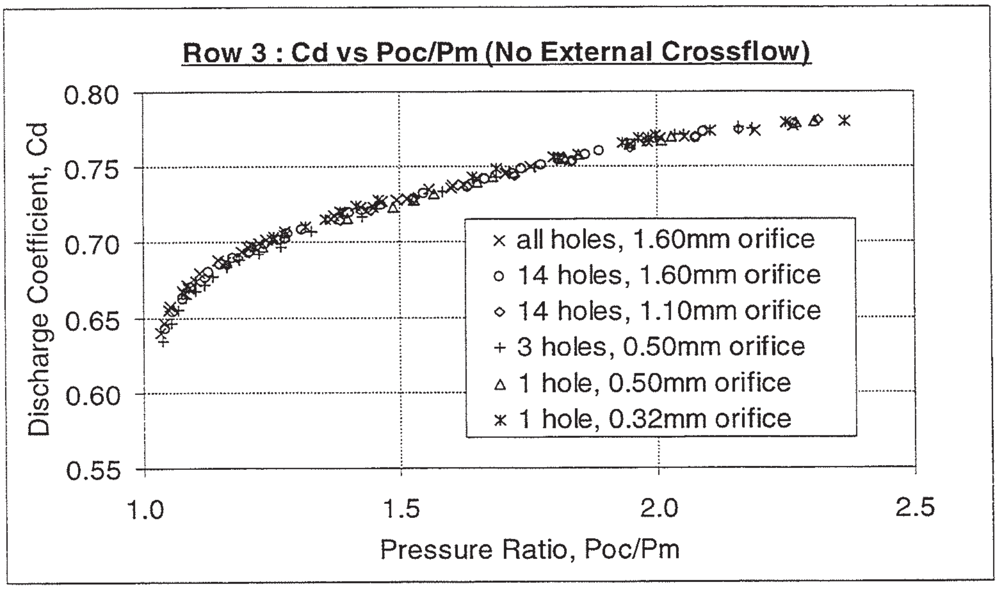

Figure 5 : Consistency in C_d Measurement Using Different Metering Orifice Plates

The importance of the assumed/determined external (i.e. blade surface) Mach number, M , should be realised as this both defines the flow and sets the external static pressure. Figure 6 illustrates the error in calculated C_d that can arise due to the nominal Mach number being in error by +/- 0.02. For a measured M, this corresponds to a static pressure measurement error of only 0.50%. Consequently, great care was taken to accurately determine the external Mach number.

The blade surface Mach number distribution was measured in the CHTT and compared to the engine designers computational prediction. It was difficult to obtain accurate Mach number measurements in the stagnation region because of the poor spatial resolution of surface pressure tappings. As a result, in this region, the Mach number was interpolated from Rolls-Royce CFD predictions, as these had a greater number of data points around this critical, leading edge region. Away from this region the measurements were found to be in excellent agreement with the designers prediction.

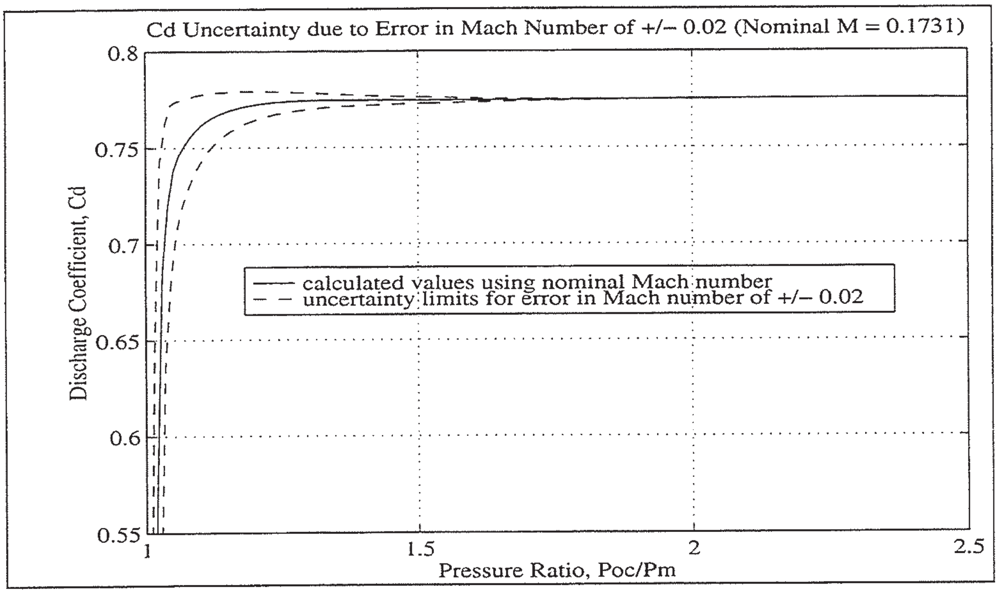

Figure 6 : Variation in C_d due to Error in Assumed External Mach Number

Those pressures required for the determination of the film cooling hole C_d with external flow present - namely those upstream of the cascade and within the cavity - were initially recorded independently and relative to atmosphere. However, defining the pressure 'sensitivity' as the ratio of the percentage error in C_d due to a pressure reading error to the percentage error in the pressure reading, it is found that although this method of pressure measurement is sufficiently accurate at high Mach numbers (sensitivity = 1.01 at $M = 0.98$) it is not at low Mach numbers (sensitivity = 26.3 at $M = 0.01$), as illustrated in Figure 7. Consequently, in the current series of experiments the coolant and mainstream pressures are measured differentially, leading to a dramatic reduction in sensitivity at low Mach numbers (sensitivity less than 0.50 for all M).

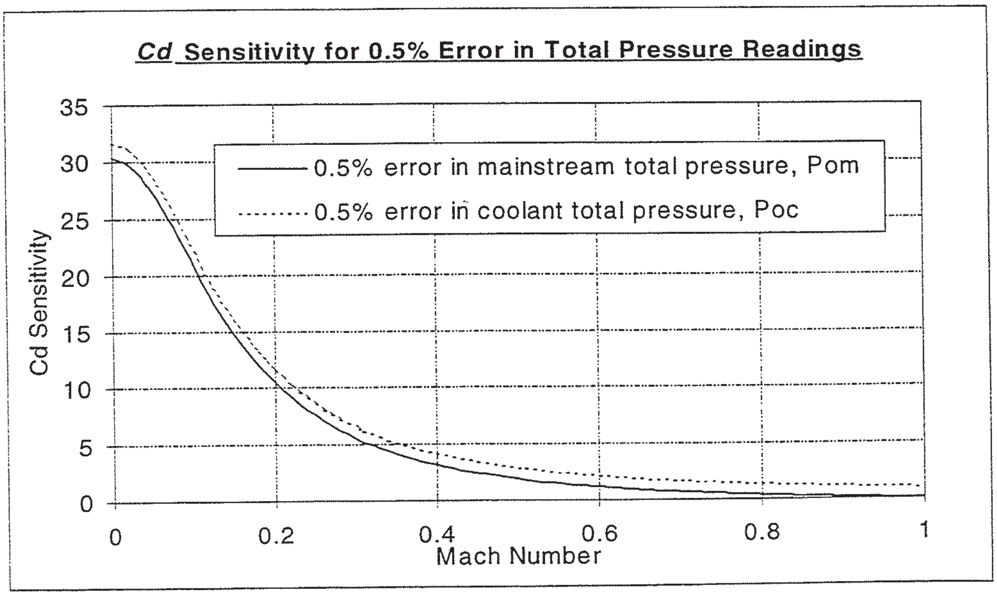

Figure 7 : C_d Sensitivity (= $\frac{\text{\% error in calculated Cd}}{\text{\% error in measured pressure}}$) to Errors in Total Pressures

6.2 Measurements with External Crossflow

The discharge coefficients of rows and groups of film cooling holes were experimentally determined over a range of cavity-to-external pressure ratios, incorporating the design case. Examples of data obtained are shown in Figures 8 and 9. It can be seen that, as expected, the mean C_d of the open holes when the flow is choked (i.e. $PR > \sim 2$) is unaffected by whether or not there is crossflow at the hole exit, that altering the coolant-to-mainstream density ratio - by changing from air to foreign gas coolant - has no effect on the hole C_d at the same PR and, generally, that there is a significant difference between the C_d of the holes with and without crossflow at typical engine pressure ratios.

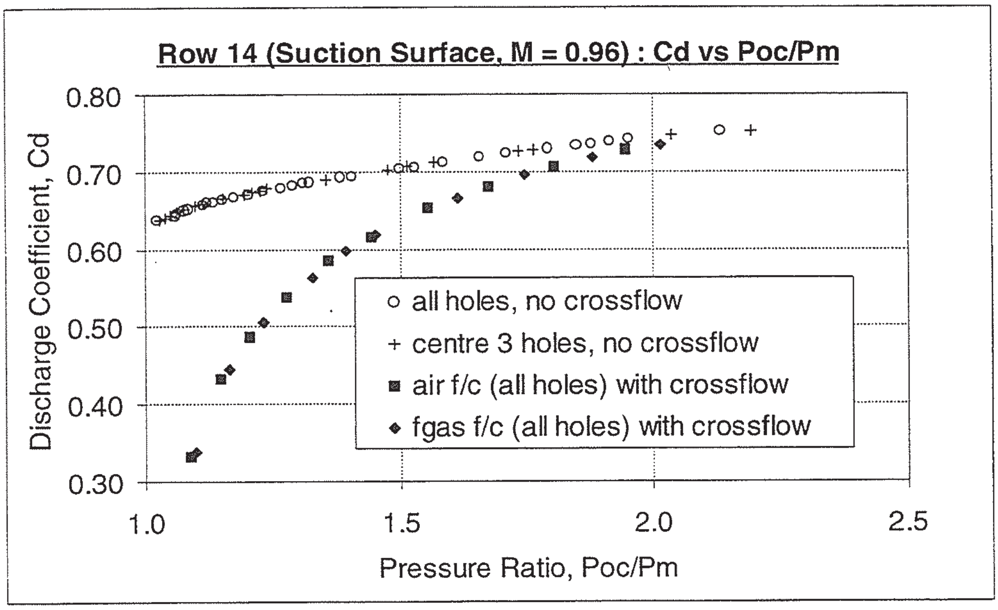

Figure 8 : Cd vs PR Data - both With and Without External Crossflow - Illustrating 'Classical' Behaviour

The data shown in Figure 8 demonstrates what might be termed as the 'classical' behaviour, in that the discharge coefficient with crossflow is lower than the non-crossflow case for all pressure ratios. In contrast, the data presented in Figure 9 illustrates the case where, at the same pressure ratio, the with-crossflow data is initially lower than the non-crossflow data, but becomes higher than the non-crossflow case as the pressure ratio increases and then tends to the same value as one approaches choked flow. This is termed the 'cross-over' phenomenon. Out of the thirteen rows of cooling holes studied, nine rows demonstrate some level of cross-over. The extent of the cross-over appears to depend upon the external Mach number and the orientation of the hole to the flow. It should be noted that there are a number of instances in the relevant publications where the data presented shows a tendency for the afore-mentioned cross-over effect - see, for example, Rhode, Richards and Metger (1969), Hay, Lampard and Benmansour (1983), Andrews and Sabersky (1990) or Hay, Lampard and Khaldi (1994) - although the authors usually fail to discuss this.

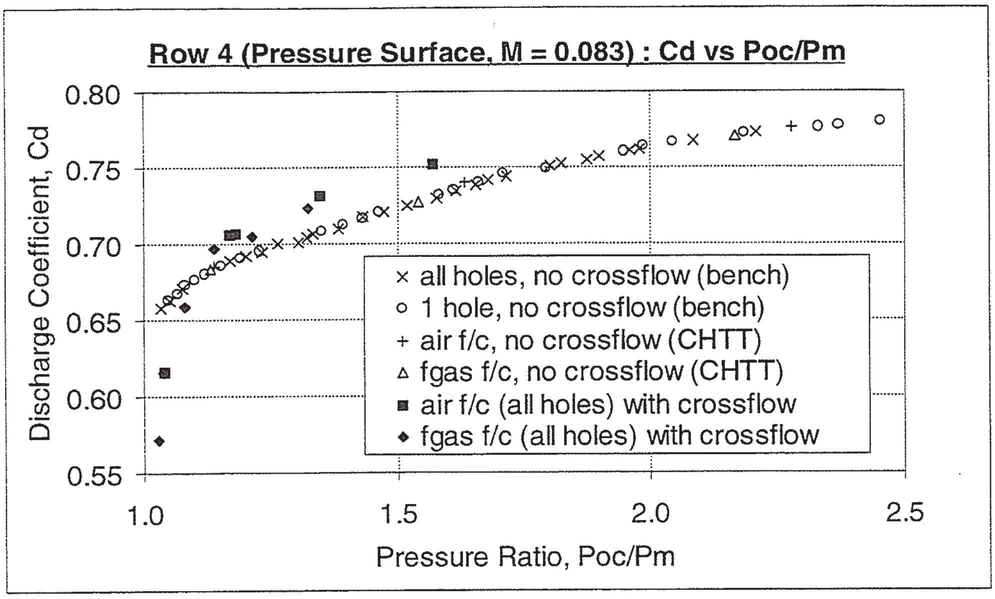

Figure 9 : Cd vs PR Data - both With and Without External Crossflow - Illustrating the 'Cross-over' Effect

The discharge coefficients of cylindrical film cooling holes have been measured on a fully film cooled NGV blade in an annular cascade tunnel, under engine representative conditions, for two coolant-to-mainstream density ratios over a wide range of pressure ratio that encompasses the design value. The discharge coefficients of the film cooling holes are significantly affected by external crossflow - by as much as 12% at typical engine pressure ratios - although the effect can be to either increase or decrease the C_d depending on the geometric and flow conditions present. This is of great interest and importance to aero-engine designers.

The cross-over phenomena is currently under further investigation.

7 Acknowledgements

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