

*12th Symposium on Measuring Techniques  
for Transonic and Supersonic Flow in Cascades and Turbomachines*

Prague, Czech Republic, September 12-13, 1994

---

**RESEARCH ADVANCES FOR TURBOMACHINERY APPLICATIONS  
USING PARTICLE IMAGE VELOCIMETRY**

M. Funes-Gallanzi, P.J. Bryanston-Cross  
University of Warwick  
England

# RESEARCH ADVANCES FOR TURBOMACHINERY APPLICATIONS USING PARTICLE IMAGE VELOCIMETRY

M. Funes-Gallanzi & P. J. Bryanston-Cross  
Department of Engineering  
University of Warwick, U.K.

## Abstract

The work presented in this paper deals with means of making spatial PIV measurements at high speeds more accurate. The PIV technique is now a well developed technique for fluids research. Further, a move to three dimensional measurement of the velocity field, to bring the accuracy of the estimates up to the standards demanded by aerodynamicists, is the next step in the development of the technique. At high speeds, the seeding densities in regions of interest, such as wakes or boundary layers, are not high enough to enable the use of frequency techniques; which in any case cannot capture small scale information.

Essentially, a diffraction limited optical component has been used to provide aberration free particle images. Secondly, the sensitivity of the CCD cameras has been increased with the use of bespoke tuning for laser operation. Thirdly, it has been found that it is possible to record double, with a lower displacement than in the past, particle images using a pulsed laser. Fourthly, the particle data can be automatically analyzed using a software program. Finally, since the data is recorded in stereo, it is possible to obtain instantaneous 3-D particle whole field velocity maps. This 3D velocity estimate is made accurate to the order of  $30\mu\text{m}$  with the help of high precision measurements of position in all three dimensions; exploiting the intensity information in the data. Such an intensive amount of processing is only feasible due to the large data compression which is possible when using the spatial approach to PIV analysis.

The aforementioned advances enable the instantaneous quantitative whole field visualization of high-speed unsteady regions of flow in turbomachinery; which has been successfully compared with a full viscous calculation. This work represents the first such measurements to be made in all three dimensions with such accuracy at realistic speeds for turbomachinery research purposes.

## 1. Introduction

For many years several groups of people have been researching optical methods for the extraction of three-dimensional flow. Throughout the last decade several techniques have been developed. Laser Doppler Anemometry (LDA) using a continuous wave laser to temporally scan a single spatial point through the flow field is the most commercially exploited technique. At face value the application of LDA seems straightforward. However, as the study of fluid dynamics has evolved, from being able to solve and measure the steady state problem towards the reality of three-dimensional turbulent unsteady flows, the inherent limitations of temporally averaged data have become more apparent. Alternatively, whole field flow visualization methods such as holographic interferometry have failed to deliver a true solution to this problem for two main reasons. Firstly, the complexity of the holographic system has made its application specific to the experimental case in question. Secondly, the complexity of deconvolving an integrated refractive index field adds a further limitation to the general application of the technique.

The more recently evolved Particle Image Velocimetry (PIV) simplifies the acquisition of whole field data to a photographic

and/or CCD imaging process, but like LDA, still requires considerable mathematical processing of data. The PIV data to date has, as yet, inherently been of a two-dimensional nature. In order to extract the out-of-plane component various authors have concentrated on the use of triangulation. However, if only triangulation is used, the relative error in the out-of-plane direction is of the order of three times larger than in the x-y direction. This effectively makes the technique of no practical industrial use where three dimensional flow obtains. Furthermore, both the relative and absolute co-ordinate systems remain basically unrelated as the measurements obtained by triangulation are relative between pairs of particles and have no relation to a frame of reference.

The experiments described in this paper provide a fundamental shift to 3DPIV and addresses the limitations. With computer automated particle image processing, it is then possible to construct the vectorial distribution of position and velocity of the flow field by a combination of techniques.

## 2. Experimental Systems

Part of the testing was carried out in the ILPC at DRA Pyestock and at the new transonic facility of the Department of Engineering, University of Warwick. The ILPC is a short duration facility designed to allow high quality heat transfer and aerodynamic measurements to be taken for a full-size annular cascade of turbine vanes. The use of this technique for turbomachinery measurements was pioneered by Schultz et al, (1973). The Pyestock facility is described by Brooks et al, (1985).

The Warwick University Transonic Blowdown facility is a transient tunnel surrounded by an Anechoic chamber. The initial function of the test cell had been the evaluation of noise levels at the exit of a transonic jet. The facility is described in detail by Funes-Gallanzi, (1994b).

Images employing a CCD camera and K2 diffraction limited optics arrangement were recorded; using a frame grabber and related electronics. The particle images shown were then processed using a software package developed at the University of Warwick called APWin. APWin performed its processing under Windows on a PC. The results were then downloaded via Ethernet onto the departmental Sun system. A second stage of processing followed, at the end of which a revised velocity vector plot was produced using the Matlab environment.

## 3. PIV Experimental Techniques

The basic aspects of PIV measurement for transient transonic facilities, such as the DRA ILPC, have previously been reported in the literature (for instance Bryanston-Cross et al 1990, 1991 and Funes-Gallanzi et al 1994a). Therefore, only the areas of recent development shall now be covered.

### 3.1 Imaging System

The solid state PIV system currently being tested, includes a computer-controlled high-sensitivity stereo CCD camera arrangement, together with image analysis tools which allow the user to assess the results of the test within a half hour period. The whole system being externally triggered by the rotor, to allow the investigation of stator-rotor interactions at a given stator/rotor relative position. In fact, the system has been recently successfully tested at Mach 1.1. As soon as the ILPC is available, this video system will be tested at DRA Pyestock. It is designed to cope with the exacting requirements demanded by the stator/rotor interaction research currently under way. It is schematically described in figure 6 : and fully described by Funes-Gallanzi et al, (1994b); which also contains a description of the latest developments of the APWin spatial approach PIV software.

### 3.2 Out-of-plane Velocity Measurement

The use of the stereo approach enables the measurement of the relative velocities within the region of interest illuminated by the light sheet. Two stereoscopic configurations can be used. The angular method has the optical axis of both cameras intersect the illuminated field containing some angle thereby. The translation method has the optical axis of both viewing cameras parallel to each other and perpendicular to the light sheet. According to Gauthier & Reithmuller (1988), the angular displacement method is more accurate. However, the required depth of field is larger than in the translation method. For a given magnification, a large depth of field can only be obtained by increasing the f-number; i.e. decreasing the light sensed by the CCD array. This is also a limitation on the widest practical angle between the cameras. In practical applications such as turbomachinery, the area of interest (boundary layer, wake, etc.) is generally quite small in absolute terms. This fact, coupled with the need for a realistic stand-off distance using diffraction-limited optics, means that only the angular method can be reliably used yielding a meaningful overlapping area.

The 3D approach allows the measurement to be more accurate than previous two-dimensional estimates. However, if only the geometric approach is employed, the accuracy in the out-of-plane direction is of the order of 3 times that in-plane. For turbomachinery, the out-of-plane velocity component is typically and order of magnitude smaller than in-plane. Consequently, the combination of these two effects means that the errors involved in estimating the out-of-plane component are roughly of the same magnitude as the velocities being measured.

### 3.3 Position estimation

For calibration purposes, an England finder graticule of 1x3 In. is used as it provides accurate and detailed information; which can be used for position estimation, and PIV scaling.

Once initial results were obtained it became clear that, given the high errors in the out-of-plane direction, the stereoscopic approach to PIV would remain impractical unless a way was found to increase the accuracy in the z-direction.

In general, one way to increase the accuracy is of course to increase the angle subtended by the two cameras. However, this is often not possible in real applications as facilities have to be adapted to, which often were not particularly designed for visualization purposes, the depth of field required increases as the angle increases, and are further restricted by the need for simplicity and economy. Furthermore, as the angle increases the absolute spatial errors also increase; thus denying some of the increase in accuracy.

As mentioned earlier, the spatial approach to PIV has as one of its advantages the ability to apply fairly intensive processing to the PIV pairs found; thanks to the large data reduction involved.

Applications involving 3DPIV experiments at transonic speeds represent a challenge which requires novel high accuracy position estimation techniques, these being split between those required for absolute position estimation and relative position estimation. A well known and quite difficult problem in conventional PIV is to ensure that the data is being viewed - or at least displayed after suitable rotations/translations - orthogonally to the light-sheet. This problem is exacerbated in 3DPIV and it has been found to be impossible to determine absolute position as accurately as required; under realistic experimental conditions. Therefore, a novel approach has recently been presented (Funes-Gallanzi et al, 1994c) of making an initial estimate of the parameters on-site and subsequently, by employing two views of a reference image and a mixture of techniques implemented in the REGISTER software package, go through a second corrective stage; thus improving the accuracy of the estimates up to the required standard.

It is of crucial importance to find the absolute position of the frame of reference provided by the focal distance. In this way, the relative position (and therefore velocities) of particle pairs can be firstly found relative to this point, and then its absolute position in space can be ascertained.

Error analysis for stereoscopic PIV has been investigated for the geometric parameters as shown by Prasad & Adrian (1993). However, a comprehensive investigation, even for the geometric aspects, including the effects of registration error was not covered. Particle size, which is given by the speed of flow under consideration, influences accuracy in two distinct ways. Firstly, its image size relative to the pixel size determines the maximum spatial resolution. By exploiting grey level information, it is possible to achieve sub-pixel (of the order of 1/10th of a pixel) accuracy with no bias. Secondly, varying particle sizes, electronic noise, image smearing, etc. contribute to a random component. By employing the image intensity approach to PIV, it is possible to discriminate particles according to their size and to deal with electronic noise, thus reducing the impact of random errors on the accuracy of the measurements. Image smearing is sometimes present in DPIV data and not accounted for; when the CCD sensor dimensions are not matched by the image grabber (commonly a 512 by 512 grabber is used while the image is sensed by a 768 by 576 pixels array). The description of how the absolute position of the frame of reference can be accurately described is split into two distinct parts:

The first part involves the calibration of the solid state cameras to suit the dynamic range and ensure that their light response is identical. This is achieved with the help of a Baum chart and USAF 1951 resolution chart. This stage is of crucial importance, if grey level information is to be used in the data analysis stage. Thus, although some authors consider the use of digital cameras in PIV to be merely an extension of the technique, if grey level information is also of interest, more general solid state information and calibration is required. The determining factor is the effective number of grey levels employed (or in other words the contrast). Therefore, to tailor the dynamic range of the cameras to the experimental light conditions yields large improvements in accuracy.

The second part consists of placing an England finder and taking a picture with the stereo PIV arrangement. An England finder is a graticule containing labeled squares of 1mm by 1mm which are each split into four regions and contain the square label in an inner circle. From these images, it is possible to make a first

attempt at reconciling all views of the finder; using the measured parameters of location and distances of the cameras to region of interest et cetera. In the ideal case, if no errors were present, these would result, after a transformation to a common viewing point, in identical images being formed from all cameras. However, this is in general not the case. Therefore, various characteristics of the images are used to minimize these errors and produce corrected estimates which can locate the frame of reference with a high degree of accuracy. The chosen common viewing point is given by a plane orthogonal to the light-sheet at the average focal distance of the viewing cameras.

The system was successfully tested at the new Warwick University transonic blowdown facility; using an England finder as the reference image. The translation error, after software correction, in the x and y-direction was limited to no more than 1 pixel for translations of up to 50 pixels. On the other hand, the rotational errors measured in pixels versus degrees were of the order of 1 degree for a typical experimental error of 5 degrees; the rotational error of one camera with respect to the other being considerably smaller of the order of half a degree due to the use of a spiral FFT approach in that case.

The whole process is involved and time consuming but can be adapted to virtually any set of test conditions. Thus, the price paid for this increase in accuracy in PIV measurements, is a somewhat longer and rigorous setup stage before measurements are actually taken. A new software package to automatically carry out corrections to the initial estimates on the basis of the image information is currently under further development so it can be integrated with the existing PIV software. The main advantage of such an approach is that, particularly in an industrial environment, it is often not possible to make physical measurements with great accuracy; given the facilities and time available. Therefore, a second corrective stage off-site makes possible highly accurate PIV measurements; even in hostile industrial conditions.

Turning now to the issue of relative position estimation, a digital image is a spatial, intensity and temporal quantized representation of a real-world scene. The precise representation of position is critical to the successful extraction of velocity data from a PIV image. For a solid state PIV implementation to approach the precision of a photographic image, accurate sub-pixel position estimates are necessary. Without such an approach, the large size and number of required digital images present currently un-surmountable problems of registration, data volume and throughput.

Studies on the nature of digital images rarely take specific notice of the effects and opportunities offered by intensity quantization. Normally, only spatial quantization is considered; though occasionally the strong effect which the number of bits per pixel has is recognized. A full description of the combined effects of spatial and intensity quantization has been reported (Bruckstein, 1987 for instance) but in a statistics context and without regard to the geometry or specific image content.

In view of this, an approach is currently being developed based on the concept of position equivalence classes, referred to as "locales", and first developed by Havelock (1989). Some independence from image form is attained by considering a general gray scale shape. A natural consequence of the concept is the definition of an optimally precise position estimate given by the centroid of the locale. It can thus be shown that useful dynamic range is far more important than pixel size for obtaining geometric precision when data volume is constrained. Furthermore, it can also be shown that the best way to allocate spatial and intensity resolution for a digitizing scheme subject to data vol-

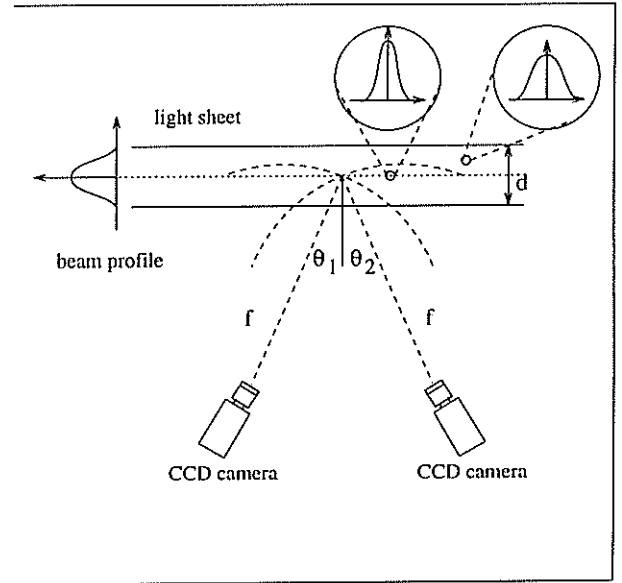


Figure 1 Gaussian form dependence on position

ume constraints is to allow only enough spatial resolution for effective detection and recognition and employ all remaining data capacity to maximize the available intensity resolution.

Typically, if the irradiance is given by:

$$I(r) = \frac{P}{2\pi\sigma^2} e^{-\frac{r^2}{2\sigma^2}}$$

where P is the input power, the beam diameter is given conventionally by  $d=4\sigma$ , which means that over the interval considered ( $0 \leq r \leq 2\sigma$ ) only 86% of the power is contained. The rest provides stray illumination and leaks through the diffraction limited optics to lower the contrast. Thus, the CCD cameras have to be tuned to the prevailing light conditions to yield the greatest available contrast.

This concept can be applied to PIV as follows: a fairly realistic representation of the image of a mono-dispersed particle (assuming it is a point source) can be provided by a Gaussian form.

$$E(x,y) = A e^{-\frac{((x-x_0)^2 + (y-y_0)^2)}{2\sigma^2}}$$

The particle image is located at  $(x_0, y_0)$ . The Amplitude A and width  $\sigma$  are the only parameters of its circularly symmetric shape.

In fact, for sub-micron particles imaged through diffraction-limited optics, it is the point spread function which is imaged onto the CCD chip face. The diameter of the PSF is then given by (Adrian, 1991):

$$d_s = 2.44(1 + M) \frac{\lambda f}{D}$$

where M is the image magnification,  $\lambda$  denotes the illumination wavelength, f is the focal length and D the lens diameter. Further, the axial distance over which the image is in sharp focus is defined as:



Figure 2 PIV image of a field

$$\Delta z = 4(1 + M^{-1})^2 f^2 \frac{\lambda}{D^2}$$

The above definition determines the difference in axial positions of positive and negative defocus producing a reduction in the central peak of the PSF to 80% of its peak value. This definition is conservative and it does not correspond exactly to the imaging characteristics of the PIV system, but it does serve as a guide; the exact effective depth of field being currently under investigation. The above equations can be used to show that in order to resolve sub-micron particles, the corresponding depth-of-focus is of the order of 3mm.

Many algorithms can be found to estimate the position of the centre of this object. For instance, a centroid estimate gives the correct answer in the absence of noise and quantization. In the presence of spatial quantization there appears a skew error which can be accounted for in a unique mapping. In the presence of intensity quantization, as is normal for digital images, there is not a unique but a set of possible object positions. These regions are referred to as locales and their size determines the uncertainty in object position. There is no equivalent situation in non-digital imagery.

With reference to the schematic diagram in Figure 1, it is possible to establish what these two parameters,  $A$  and width  $\sigma$ , in turn depend on.

Note that in order to simplify the discussion, the centre of the Nd/YAG sheet and the centre of of focus are made co-incident and the depth of view is made somewhat larger than the width

of the light sheet. Thus, the amplitude can be seen to depend on the  $z$ -position of the particle and vary according to the change in intensity of the laser over the depth of the region of interest. On the other hand,  $\sigma$  varies approximately linearly according to the  $z$ -position in relation to the position of focus. This focal length can be quite accurately calculated for a given objective in the case of the K2 diffraction limited optics. As the particle moves out of focus, so  $\sigma$  will vary. Thus if the ratio of  $A/\sigma$  (referred to as the depth ratio) is considered, it is directly and linearly related to the focal position.

This technique has two major advantages. Firstly, it provides three measures for the  $z$ -component. Two from the  $A/\sigma$  ratio from each image in a stereo case, and the third from triangulation. Just as importantly though, it provides a way in which these relative velocity measurements can be related to an absolute frame of reference. The depth ratio will exhibit a maximum where the particle is in line with the focal length of the lens, and will then tail off as a particle moves in front or behind this position. Thus, the system has a symmetry about this focal length leading to an ambiguity in the measurement of the depth ratio. In order to account for it, triangulation needs to be used as well. Thus if a particle pair lies in a equidistant positions from this axis, the ratio will be equal but triangulation will show one to lie ahead of the other, thus enabling the data to be unscrambled. In the case of turbomachinery, the second camera is actually not required as assumptions can be made about the direction of travel in the out-of-plane component.

The system can be calibrated by plotting all the depth ratios and finding the maximum; which is the frame of reference as it is co-incident with the focal length. The relationship between the depth ratio and absolute depth can also be ascertained from this plot by introducing the width of the light-sheet and thus deduce the scaling factor. The potential accuracy of this approach has been estimated at 20-30 $\mu\text{m}$  in all three directions. This is arrived at by considering an effective grey level dynamic range of 100-150 grey levels, and assumes that all parameters have been estimated by the use of REGISTER as previously discussed.

#### 4. Results

Preliminary results are shown in Figures 3 and 4 and 5. Particle data was obtained from the data field shown on Figure 2; which consisted of 1280 particles extracted from a transonic wake flow at 1.1 Mach. The particle data was firstly plotted in the form of a histogram to verify the overall characteristics of the visualization. As expected, the histogram shows a peak which corresponds to the calculated maximum, considering the peak power and particle size. Secondly, the particle density falls as  $\delta z$  increases, due to the second particle image being more likely to fall out of the light sheet. Thirdly, there is an abrupt drop at a depth factor of about 60, corresponding to a light sheet thickness of 1mm. Within those overall picture, however, three distinct peaks can be observed, these correspond to, firstly and rightmost, the 0.5 microns styrene particles used for this experiment. In the middle, overlapping particle pairs in regions of low velocity can be seen and treated separately. Lastly, the third peak corresponds to low-intensity noise erroneously treated as particles.

A pair, selected from the single-particle population, was plotted to confirm the broad nature of the particle images. The particle position was then calculated to sub-pixel accuracy by a combination of data intensity calculations and depth ratio estimates; as shown in Figure 4. This new estimates were in agreement with the previously calculated data. The estimated accuracy was in the order of 30 $\mu\text{m}$ ; an improvement on previous estimates but not as high as can be expected. The software, currently im-

plemented in MATLAB, makes use of the output from APWin to provide an initial estimate of particle position and pair velocities, which is then improved upon by the abovementioned techniques. Further research is currently under way to fully automate the process, include it in the APWin environment and gain more experience with both the locales approach to sub-pixel estimation and the statistical properties and experimental characteristics of the depth ratio approach previously described.

Two further advantages of this approach are that normally some astigmatism is present in the particle data. This can be removed by applying the techniques described. Secondly, particles which are close together and would normally be erroneously interpreted

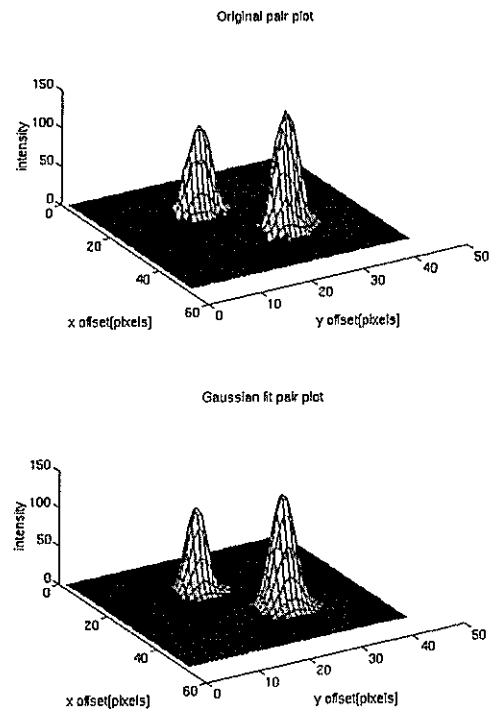


Figure 4 MATLAB plots of a particle pair in grey scale

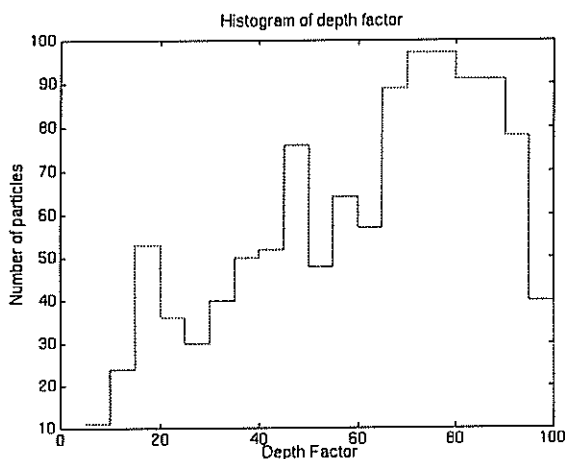


Figure 3 Depth factor Histogram

as one can be recognized and treated correctly, as shown in Figure 5, where such a case is shown. The bounds of the depth ratio are defined by the power profile on the lightsheet, the particle size and shape, and by the prevailing light conditions. Therefore, particle of a different size and/or shape to those representing the seeding can be recognized and treated accordingly. Thus, the presence of stray particles in the flow does not influence the analysis.

The accurate measurement of relative velocity in three dimensions presents some formidable problems. In particular, the whole approach requires the centre of focus to lie within the light sheet. This has proven to be particularly difficult in industrial applications. A reliable way to overcome this problem has been to make use of an oscilloscope - by looking at a single video line - to focus rather than sight. The locales approach, while more accurate than centroid estimation or Fourier phase estimation, is still under active development. Calibration of the cameras so they are not saturated at the focal length and other calibration requirements mean that although the accuracy is increased, the complexity in performing these experiments also increases.

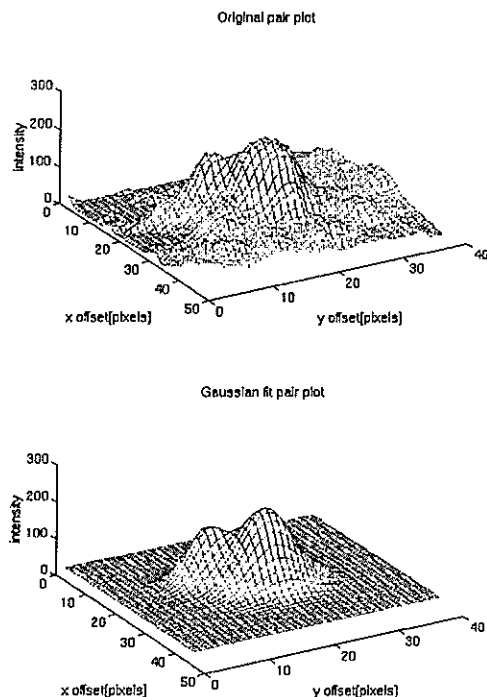


Figure 5 MATLAB plots of overlapping pair in grey scale

## 5. Conclusions

The techniques described deal first of all, with the successful determination of absolute co-ordinate information, in an industrial environment, to an accuracy of  $\pm 2$  pixels for the experiments described. This approach involves a second corrective software-implemented corrective stage, after all parameters have been physically measured.

Secondly, the grey level information was employed to provide an estimate of the three dimensional relative position with an accuracy of  $30\mu m$ . The relative and absolute position estimates could then be combined.

Three dimensional measurements of the particle pairs were successfully accomplished and so 3D velocities were estimated. The software package APWin was employed to obtain the velocity information. A further program was used to plot and estimate the three dimensional position of the particle pairs found, exploiting not only triangulation but also grey level information. Lastly, the technique was extended to allow for the recognition of close overlapping particle pair data and the removal of astigmatic effects. The development of these techniques, and the measurements obtained, mean that the short term target of achieving 1% accuracy for 3DPIV measurements at high speeds, for complex unsteady flow turbomachinery applications, is now possible.

Instantaneous aerodynamic measurements on a fully annular transonic turbine nozzle guide vane, at three engine representative conditions, will be attempted in the near future using the same equipment as described in this paper.

PIV is applicable to either a compressor or turbine, in either the rotor or stator cases. It is a quick, cheap and reliable method for

mapping an unsteady transonic flow field on full size turbomachinery at realistic conditions.

The measurements made in this paper represent an almost instantaneous whole-field picture of an unsteady transonic flow at a realistic stand-off distance; which could not have been made using conventional measurement techniques. Therefore, the techniques described in this paper open the way for such instantaneous whole-field measurements to be made of such complicated effects as stator/rotor interactions for the first time.

## 6. References

- Adrian, R.J., "Particle-Imaging Techniques for Experimental Fluid Mechanics", *Annual Rev. Fluid Mech.*, Vol. 23, pp. 261-304, 1991.
- Brooks, A.J. et al, "The Isentropic Light Piston Cascade at RAE Pyestock" AGARD-CP-390, 1985.
- Bruckstein, A.M., "On optimal image digitization", *IEEE Trans. Acoustic Speech, Signal Processing*, vol. ASSP-35, pp. 553-555, Apr. 1987.
- Bryanston-Cross, P.J., Epstein, A. "The application of sub-micron particle visualization for PIV (Particle Image Velocimetry) at transonic and supersonic speeds", *Prog in Aerospace Sci.* 27, 1990, 237
- Bryanston-Cross, P.J., Haragama, S. P., Towers, C. E., Towers, D. P., Judge, T. R., Hopwood, S. T. "The Application of Particle Image Velocimetry (PIV). In a Short Duration Transonic Annular Turbine Cascade", Submitted to the American Society of Mechanical Engineers, for the 36th ASME International Gas Turbine & Aeroengine Congress, Orlando, Florida, USA, June 3-6 1991
- Funes-Gallanzi, M., Bryanston-Cross, P.J., Chana, K.S., "Wake Region Measurement of a highly three-dimensional nozzle guide vane tested at DRA Pyestock using Particle Image Velocimetry", *ASME*, 94-GT-349, The Hague, June 13-16, 1994a.
- Funes-Gallanzi, M., "Flow Density Visualization Enhancement to the Warwick Solid State 3DPIV System", *VIIth Symp. on Appl. of Laser Techniques to Fluid Mech.*, Lisbon, 1994b.
- Funes-Gallanzi, M., Bryanston-Cross, P.J., & Udrea, D.D., "High Accuracy Techniques Applied to the Extraction of Absolute Position Estimates in a 3DPIV System", *VIIth Symp. on Appl. of Laser Techniques to Fluid Mech.*, Lisbon, 1994c.
- Gauthier V. & Reithmuller M.L., Application of PIDV to complex flows: Measurement of the third component. In VKI-LS 1998-06 "Particle Image Displacement Velocimetry", Von Karman Institute for Fluid Mechanics, Rhode-Saint-Genese (Belgium), 1988.
- Havelock, D.I., "Geometric Precision in Noise-Free Digital Images", *IEEE PAMI*, Vol. 11, No. 10, Oct. 1989.
- Prasad, A.K. & Adrian, R.J., "Stereoscopic particle image velocimetry applied to liquid flows", *Exp. Fluids*, 15, 49-60, 1993.
- Schultz, D.L. & Jones, T.V., "Heat transfer measurements in short duration hypersonic facilities" AGARD-AG-165, 1973.