Paper 3

DATA REDUCTION OF WAKE FLOW MEASUREMENTS WITH INJECTION OF AN OTHER GAS

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Abstract

The data reduction method based on the conservation laws for mass, momentum and energy has been widely adopted for the reduction of wake flow measurements downstream of cascades.

The present paper shows the extension of this method to flow fields with the injection of one or more foreign gases. This occasion has been taken to work out a thoroughly revised presentation of the complete procedure including the mixture of different gases and variable temperatures. The significance of the two mathematical solutions has been analysed.

Nomenclature

Symbols:

a^*	critical velocity
c_p	specific heat capacity
h_0	specific total enthalpy
M^*	critical Mach number
$\dot{m}$	mass flow
p	pressure
p_0	total pressure
q	dynamic pressure
R	gas constant
T_0	total temperature
t	pitch
w	velocity
x	coordinate normal to the cascade
y	coordinate parallel to the cascade
β	flow angle
Θ	mass flow rate
κ	isentropic exponent
ξ_i	mass fraction
ρ	density
ρ^*	density in critical state

Indices:

A	normal to the cascade plane
C	parallel to the cascade plane
1	homogeneous flow upstream
2	homogeneous flow downstream
21	upstream of the shock
22	downstream of the shock
2y	local values in the measuring plane

1. Introduction

The flow field downstream of a plane cascade is generally a free jet and must be measured by traversing a probe parallel to the cascade. This leads to the demand for an algorithm for the reduction of the measured data into a set of homogeneous data representative for the flow field downstream of the cascade. Obviously this homogeneous flow field can be interpreted as the mixed out flow far downstream, but this cannot be realized experimentally.

The homogeneous flow values are required as representative data, which shall replace the measured data for the turbomachinery design process. This applies also for flow fields within multistage turbomachines, where a mixed-out status is never reached.

The replacement of the measured data (Figure 1) by homogeneous values requires that both types of data are equivalent with respect to all physical properties. This means the satisfaction of the conservation laws for mass, momentum and energy.

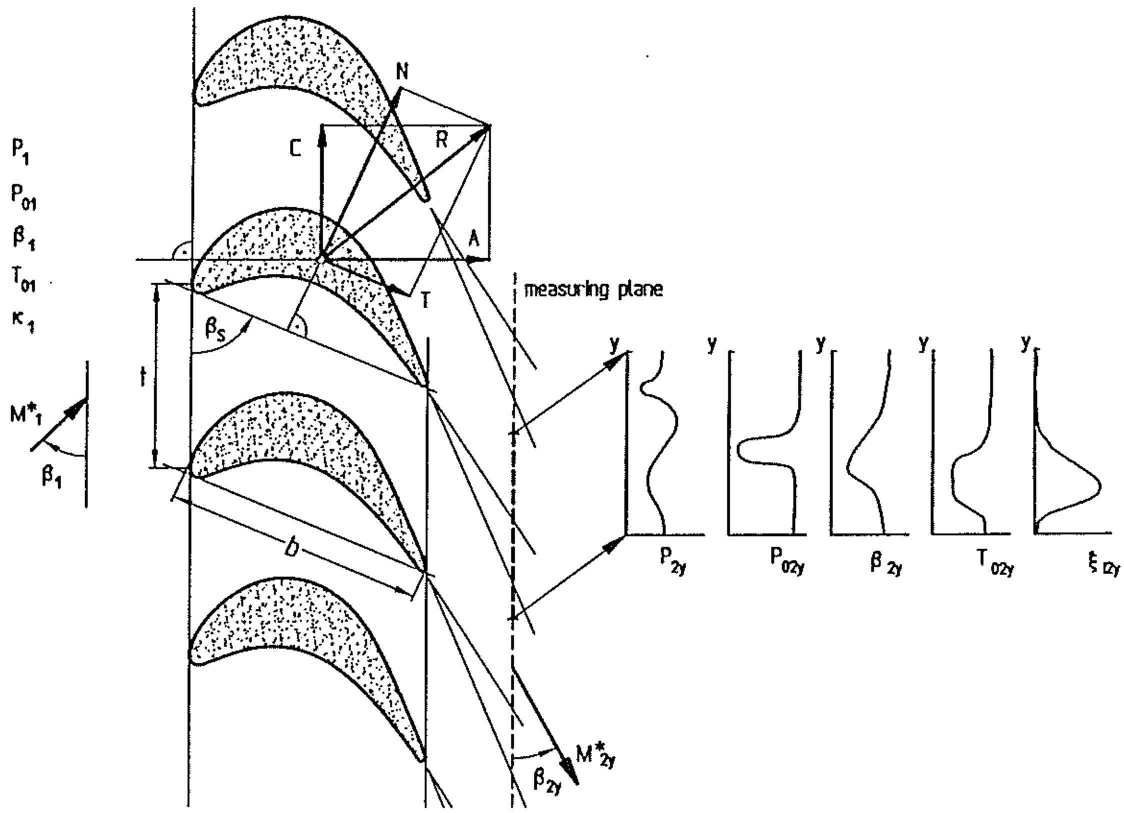

Figure 1. Nomenclature

The wake traverse method was first pointed out by Betz [3] for the experimental separation of the profile drag from the induced drag of a single airfoil tested in a wind tunnel. The application of this method modified for cascade measurements was for the first time presented by Christiani [5]. A very thorough analysis of this data reduction method for incompressible flow was delivered by Scholz [7] and since this time it has been adopted as standard method for all cascade tests. A general solution of the equations for compressible flow was derived by Amecke [1] and the extension of this method to flow fields with non-constant total temperature was later shown by Oldfield et al. [6]. In the present paper the data reduction system has been extended to cascade flow fields with the injection of one or more other gases.

This evaluation is based on the assumption that the measured wake flow is a mixture of perfect gases. The mass fraction is defined as mass of an injected component in relation to the mass of the mixture i.e. the sum of the masses of the injected components plus main flow:

$$\xi_i = \frac{\dot{m}_i}{\sum_{i=1}^{n+1} \dot{m}_i} \tag{1}$$

The mixture of a perfect gas itself is a perfect gas. Consequently we obtain for the gas constant and the specific heat capacity at constant pressure:

$$R = \sum_{i=1}^{n+1} \xi_i \cdot R_i \qquad c_p = \sum_{i=1}^{n+1} \xi_i \cdot c_{p_i} \tag{2}$$

The isentropic exponent (ratio of specific heat capacities) of a perfect gas can be expressed as function of these values:

$$\kappa = \frac{c_p}{c_p - R} \tag{3}$$

2. Mass Flux Balance

The conservation of the mass flux is established by the continuity equation[1]):

$$\rho_2 \cdot w_2 \cdot \sin\beta_2 = \int_0^1 \rho_{2y} \cdot w_{2y} \cdot \sin\beta_{2y} \cdot d\left(\frac{y}{t}\right) \tag{4}$$

The treatment becomes more general if we transform all conservation equations into a nondimensional form. Therefore eq.(4) is divided by the values of the critical state of the homogeneous flow upstream.

After introducing the (nondimensional) mass flow rate $\Theta = \frac{\rho \cdot w}{\rho^* \cdot a^*}$ and summarising the thermodynamic properties by

$$K = \sqrt{\frac{\kappa}{R} \cdot \left(\frac{2}{\kappa + 1}\right)^{\frac{\kappa+1}{\kappa-1}}} \tag{5}$$

we obtain finally the nondimensional continuity equation:

$$\frac{K_2}{K_1} \cdot \frac{p_{0_2}}{p_{0_1}} \cdot \sqrt{\frac{T_{0_1}}{T_{0_2}}} \cdot \Theta_2 \cdot \sin\beta_2 = I_M \tag{6}$$

All values on the right side can be calculated directly from the measured data:

$$I_M = \int_0^1 \frac{K_{2y}}{K_1} \cdot \frac{p_{0_{2y}}}{p_{0_1}} \cdot \sqrt{\frac{T_{0_1}}{T_{0_{2y}}}} \cdot \Theta_{2y} \cdot \sin\beta_{2y} \cdot d\left(\frac{y}{t}\right) \tag{7}$$

Chemical reactions are not treated in this study. Therefore the continuity equation must be satisfied not only for the total mass flux but also for the mass flux of each injected component ξ_i. Consequently we obtain n independent continuity equations for n injected components:

$$\left[\xi_{i_2} \cdot \frac{K_2}{K_1} \cdot \frac{p_{0_2}}{p_{0_1}} \cdot \sqrt{\frac{T_{0_1}}{T_{0_2}}} \cdot \Theta_2 \cdot \sin\beta_2 = I_{Mi}\right]_{i=1}^{n} \tag{8}$$

Accordingly we get one integral for each injected component:

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[1]) The integration range may start at an arbitrary value and should be notified correctly:

$$\frac{y_0}{t} \rightarrow \frac{y_0}{t} + 1$$

For simplification we will write:

$$0 \rightarrow 1$$

$$\left[I_{Mi} = \int_0^1 \xi_{i_{2y}} \cdot \frac{K_{2y}}{K_1} \cdot \frac{p_{0_{2y}}}{p_{0_1}} \cdot \sqrt{\frac{T_{0_1}}{T_{0_{2y}}}} \cdot \Theta_{2y} \cdot \sin\beta_{2y} \cdot d\left(\frac{y}{t}\right) \right]_{i=1}^{n} \tag{9}$$

Dividing eq.(8) by eq.(6) gives following expression for the concentration of the injected components of the mixture:

$$\left[\xi_{i_2} = \frac{I_{Mi}}{I_M} \right]_{i=1}^{n} \tag{10}$$

The application of this result allows the calculation of all thermodynamic properties of the homogeneous flow field downstream of the cascade according chapter 1..

3. Energy Balance

The conservation of the energy flux in the measuring plane is established by the energy equation. It is comprised from the continuity equation (4) and the specific total enthalpy:

$$h_{0_2} \cdot \rho_2 \cdot w_2 \cdot \sin\beta_2 = \int_0^1 h_{0_{2y}} \cdot \rho_{2y} \cdot w_{2y} \cdot \sin\beta_{2y} \cdot d\left(\frac{y}{t}\right) \tag{11}$$

For the actual application of the specific total enthalpy it sufficient to take into account only the (static) enthalpy and the kinetic energy. Therefore we can write for a perfect gas with constant specific heat capacity:

$$h_0 = c_p \cdot T_0 \tag{12}$$

This energy equation is also transformed into a nondimensional form similar to the treatment of the continuity equation in chapter 2.:

$$\frac{c_{p_2}}{c_{p_1}} \cdot \frac{K_2}{K_1} \cdot \frac{p_{0_2}}{p_{0_1}} \cdot \sqrt{\frac{T_{0_2}}{T_{0_1}}} \cdot \Theta_2 \cdot \sin\beta_2 = I_E \tag{13}$$

The integral on the right hand side of eq.(13) can be evaluated directly from measured values:

$$I_E = \int_0^1 \frac{c_{p_{2y}}}{c_{p_1}} \cdot \frac{K_{2y}}{K_1} \cdot \frac{p_{0_{2y}}}{p_{0_1}} \cdot \sqrt{\frac{T_{0_{2y}}}{T_{0_1}}} \cdot \Theta_{2y} \cdot \sin\beta_{2y} \cdot d\left(\frac{y}{t}\right) \tag{14}$$

Dividing eq.(13) by eq.(6) delivers:

$$T_{0_2} = T_{0_1} \cdot \frac{c_{p_1}}{c_{p_2}} \cdot \frac{I_E}{I_M} \tag{15}$$

Hence the total temperature of the homogeneous flow downstream can be determined in a straight forward manner not requiring the solution of the complete system of equations.

4. Momentum Balance

The momentum is a vector and consequently in a two-dimensional flow field we obtain two conservation equations:

a) Momentum flux normal to the cascade plane (axial direction):

$$\rho_2 \cdot w_2^2 \cdot \sin^2\beta_2 + p_2 = \int_0^1 (\rho_{2y} \cdot w_{2y}^2 \cdot \sin^2\beta_{2y} + p_{2y}) \cdot d\left(\frac{y}{t}\right) \qquad (16)$$

b) Momentum flux parallel to the cascade plane (circumferential direction):

$$\rho_2 \cdot w_2^2 \cdot \sin\beta_2 \cdot \cos\beta_2 = \int_0^1 \rho_{2y} \cdot w_{2y}^2 \cdot \sin\beta_{2y} \cdot \cos\beta_{2y} \cdot d\left(\frac{y}{t}\right) \qquad (17)$$

The momentum equations become nondimensional by referring them to the total pressure of the homogeneous flow field upstream.

After introducing the dynamic pressure $q = \frac{\rho}{2} \cdot w^2$ we obtain

a) for the momentum equation normal to the cascade plane:

$$\frac{p_{0_2}}{p_{0_1}} \cdot \left(2 \cdot \frac{q_2}{p_{0_2}} \cdot \sin^2\beta_2 + \frac{p_2}{p_{0_2}}\right) = I_A \qquad (18)$$

$$I_A = \int_0^1 \frac{p_{0_{2y}}}{p_{0_1}} \cdot \left(2 \cdot \frac{q_{2y}}{p_{0_{2y}}} \cdot \sin^2\beta_{2y} + \frac{p_{2y}}{p_{0_{2y}}}\right) \cdot d\left(\frac{y}{t}\right) \qquad (19)$$

and

b) for the momentum equation parallel to the cascade plane:

$$\frac{p_{0_2}}{p_{0_1}} \cdot 2 \cdot \frac{q_2}{p_{0_2}} \cdot \sin\beta_2 \cdot \cos\beta_2 = I_C \qquad (20)$$

$$I_C = \int_0^1 \frac{p_{0_{2y}}}{p_{0_1}} \cdot 2 \cdot \frac{q_{2y}}{p_{0_{2y}}} \cdot \sin\beta_{2y} \cdot \cos\beta_{2y} \cdot d\left(\frac{y}{t}\right) \qquad (21)$$

Again the integrals on the right sides comprise only measured data and can be calculated directly.

5. Solution of the System of Equations

From above analysis of the conservation laws we get finally a system of integrals [eqs.(7), (9), (14), (19), (21)], which can be calculated directly from the measured data, and a system of balance equations [eqs.(6), (8), (13), (18), (20)], which establish a system of equations for the values of the homogeneous flow. The composition of the homogeneous flow field can be derived by eq.(10) from the mass flux balances and consequently the thermodynamic properties R_2, c_{p_2}, κ_2 are at disposal according to the procedure elucidated in chapter 1.. Next the total Temperature T_{0_2} can be calculated from eq.(15).

Leaving the unknown values on the left hand side eq.(6) can be transformed into:

$$\frac{p_{0_2}}{p_{0_1}} \cdot \Theta_2 \cdot \sin \beta_2 = I_M \cdot \frac{K_1}{K_2} \cdot \sqrt{\frac{T_{0_2}}{T_{0_1}}} = \tilde{I}_M \tag{22}$$

The subsequent calculation of the critical Mach number of the homogeneous downstream flow is equal to the original procedure [1, 2] with $T_{0_2} \rightarrow T_{0_1}$ and $K_2 \rightarrow K_1$:

$$M_2^{*2} = \left(\frac{\kappa_2 + 1}{2}\right)^{\frac{2}{\kappa_2 - 1}} \cdot \frac{I_A^2}{\tilde{I}_M^2} \cdot \left[\frac{1}{2} - \left(\frac{2}{\kappa_2 + 1}\right)^{\frac{2}{\kappa_2 - 1}} \cdot \frac{\tilde{I}_M^2}{I_A^2} + \frac{\kappa_2 + 1}{2 \cdot \kappa_2} \cdot \frac{I_C^2}{I_A^2} \pm \sqrt{\frac{1}{4} - \left(\frac{2}{\kappa_2 + 1}\right)^{\frac{2}{\kappa_2 - 1}} \cdot \frac{\tilde{I}_M^2}{I_A^2} + \frac{\kappa_2^2 - 1}{4 \cdot \kappa_2^2} \cdot \frac{I_C^2}{I_A^2}} \right] \tag{23}$$

For the downstream flow angle following equation is obtained:

$$\cos \beta_2 = \frac{1}{\kappa_2 \cdot M_2^*} \cdot \left(\frac{\kappa_2 + 1}{2}\right)^{\frac{\kappa_2}{\kappa_2 - 1}} \cdot \frac{I_C}{\tilde{I}_M} \tag{24}$$

The total pressure can be calculated from eq.(22):

$$\frac{p_{0_2}}{p_{0_1}} = \frac{\tilde{I}_M}{\Theta_2 \cdot \sin \beta_2} \tag{25}$$

The preceding system of formulae is the exact solution of the conservation equations and consequently it is suited for the reduction of all wake measurements with plane cascades, as far as the conditions stipulated for the deduction of the conservation equations are valid.

The critical Mach number and the flow angle are defined only in a limited range of arguments because the radicands must be positive and value of the critical Mach number is restricted. These boundaries, calculated for different isentropic exponents, are plotted in Figure 2 as function of the momentum integrals normalized by the mass flow integral. The hatched regions indicate (for $\kappa_2 = 1.40$) impossible parameters, which may be caused by measuring errors.

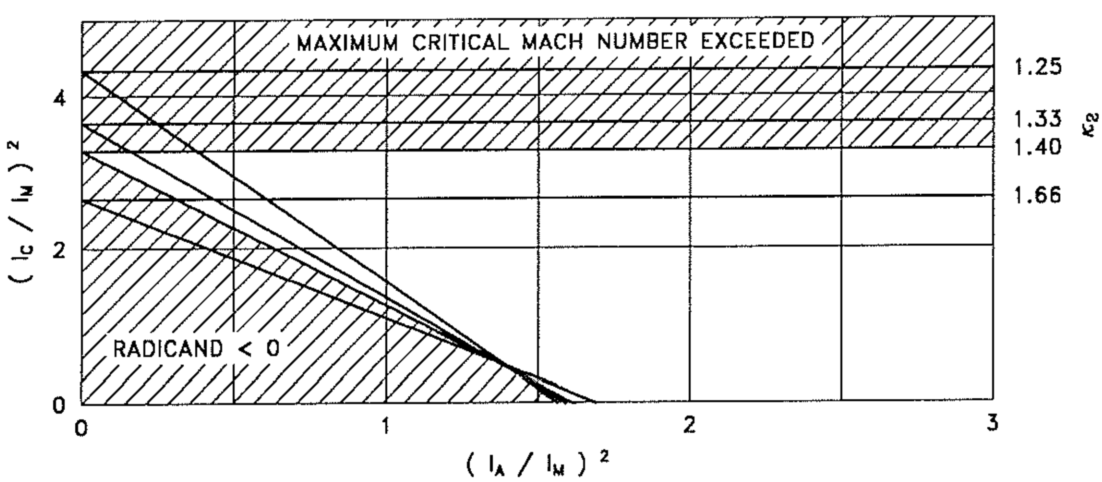

Figure 2. Range of valid arguments

In general we receive from eq.(23) four different solutions for the critical Mach number, which satisfy the system of equations. Two solutions are negative ($M_2^* < 0$) and shall not be treated further. But the two other solutions, received as solutions of the internal square root, generally represent possible physical flow conditions.

It can be shown that both solutions describe the same flow field, but upstream and downstream respectively of a shock parallel to the cascade plane. This oblique shock is the superposition of a normal shock and a uniform parallel flow field in circumferential direction (parallel to the cascade plane) and the flow values are determined by Prandtl's relation:

$$M_{21_A}^* \cdot M_{22_A}^* = 1 - \frac{\kappa_2 - 1}{\kappa_2 + 1} \cdot M_{2_C}^{*2} \qquad (26)$$

Nevertheless it is still questionable which solution of the remaining two is the representative homogeneous flow with respect to the measured flow field downstream of a cascade. Common to all can be stated only that the solution "downstream of the shock" (negative solution of the internal square root of eq.(23)) is always existent, but the other solution appears only under particular conditions:

- The flow values upstream and downstream of the shock must satisfy the second law of thermodynamics. This implies that only compression shocks (with increasing entropy) are possible. This means that the axial component of the critical Mach number must decrease while passing the shock.
- The critical Mach number may not exceed the maximum value.
- Limit loading represents the maximum velocity downstream of a cascade and it is defined by a sonic axial component of the velocity [4]. At higher velocities disturbances from downstream will not reach the cascade and affect the flow within the passages of the cascade.

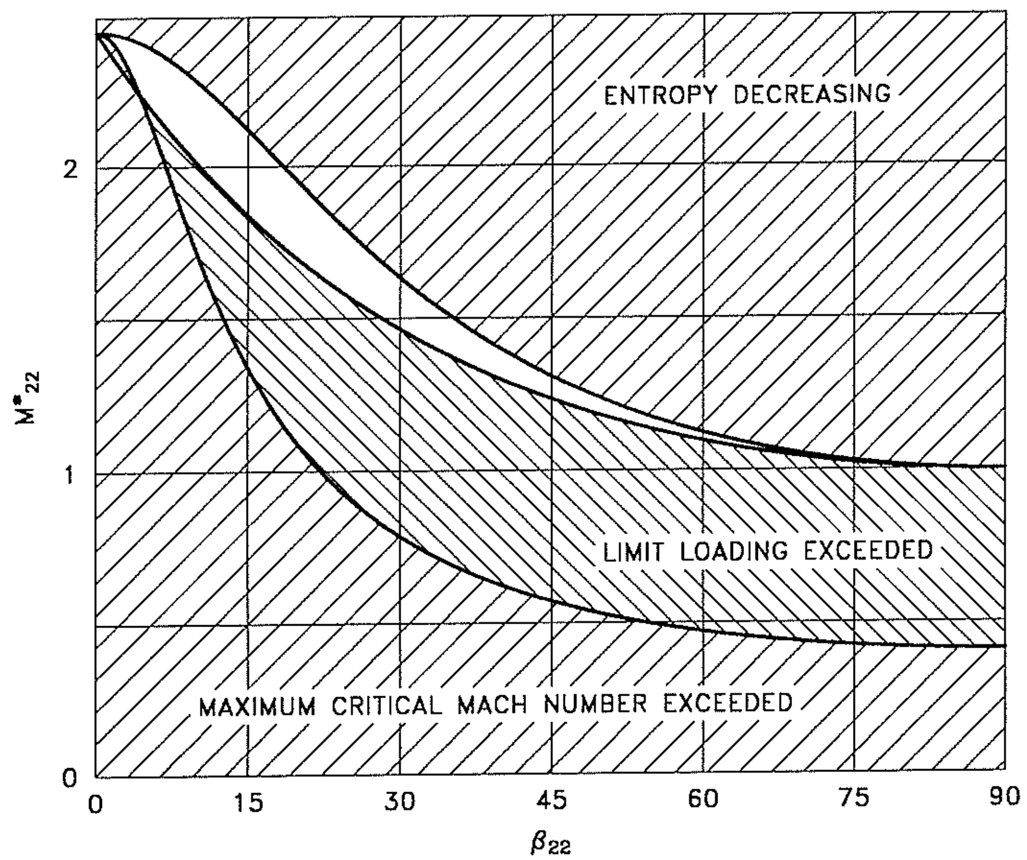

Figure 3. Regions of possible solutions "upstream of the shock" ($\kappa_2 = 1.40$)

Above mentioned conditions are plotted in Figure 3 and it demonstrates that that only for values in the small band between the entropy curve and the limit loading curve a solution "upstream of the shock" exists. This band is further restricted by the fact that with respect to unavoidable losses it is in general not possible to reach limit loading in the experimental reality.

6. Conclusions

The data reduction method for wake flow measurements downstream of a plane cascade and based on the principle of mass, momentum and energy conservation has been extended to flow fields with the injection of one or more other gases. The presented solution considers also also a variable total temperature in the wake.

The data reduction method represents the exact solution of the governing equations. The supplementary enclosure of the variable total temperature and the injection of other gases is possible in a simple straight foreward manner.

The solution exists only in a limited range of arguments. An exceeding of this range indicates faulty values (e.g. measuring errors).

The system of equations offers two possible solutions, which correspond with the physical flow conditions upstream and downstream of a shock wave parallel to the cascade. The analysis brings out that the solution "downstream of the shock" is always possible, but the solution "upstream of the shock" under exceptional conditions only.

7. References

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