Theoretical Determination of the Characteristics of Multi-Hole Pressure Probes Using Panel Methods

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1. Introduction

The problems of the pressure probe measuring technique in practice are known for a long time. Especially in small channels there is a strong influence of the probes onto the flow field. Therefore the conventional method of using this measuring technique seems doubtful as the pressure probes are calibrated in a free jet and are applied afterwards in a flow field with totally different characteristics. In a closed channel the cross section is decreased by introduction of the finite size of the probe, whereas in a free jet the influence can be neglected. Therefore there is a strong requirement to investigate the consequence of this procedure.

The major point of interest is the investigation of the interactions between a pressure probe and a closed test section with a variable cross section. With this arrangement the introduction of the probe into a cascade might be simulated. Using this basic conditions significant parameter studies can be carried out. An experimental investigation with variations of the cross section of a flow channel seems very time-intensive and cost-consuming.

In this context a method has been developed which allows to determine the changing characteristics of the pressure probe in the flow field. This method is based on the potential theory which has been implemented in the panel method.

The investigation of the changing probe characteristics in different measurement media is divided into two subtasks which shall be subsequently worked out. First the free-jet characteristics of different pressure probe types shall be reproduced by using the panel method. The results shall be compared with experimental data. In a second, more complex step the probe characteristics shall be determined for measurements in a closed channel. Some experimental results shall be compared with data.

2. Numerical Method for Calculating the Pressure Probe Characteristics

The numerical method under consideration is based on the panel method which is already well established for flow analysis of basic aerodynamic configurations in the aerospace industry. An overview of different panel codes is given by Ballmann[1]. Most panel codes refer to a method developed by Woodward [2], which has been improved by Boeing and the NASA generating the panel program PAN-AIR [3,4]. The panel program introduced in the present paper is related to a report published by H.W.M. Hoeijmakers [5]. The shape of the pressure probe can be set into direct analogy to the original task of the panel method (fig. 1). In analogy to the fuselage of an aeroplane the conical and the cylindrical part of the probe head can be regarded as a slender body. The prismatic shaft will be regarded in analogy as the wing of the aircraft.

2.1 Basics of the Panel Method

The panel method is very efficient in calculating the aerodynamic characteristics of complex configurations for subsonic flow conditions. This method is based on the linearized potential equation, which can be written as an integral equation of a singularity distribution over the surface of a body. One can regard the discretisation of the body's surface by panels with a certain singularity distribution as the kernel of this method. The local velocity vector can be determined by evaluating the strength of the singularity of each panel and by superposing this value with the parallel free-stream flow. Finally the

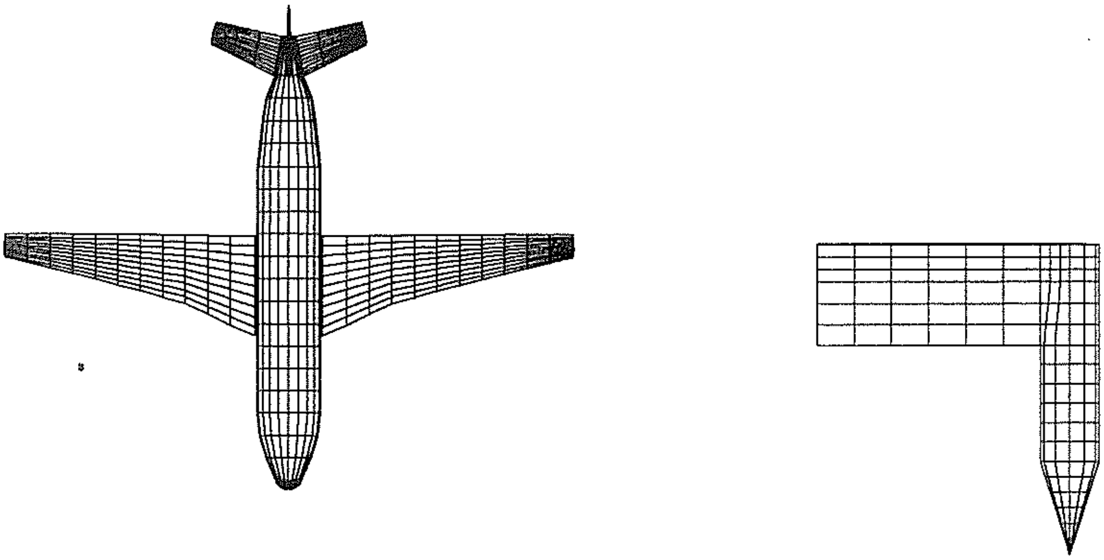

Fig. 1: Geometrical Analysis between Airplane and Pressure Probe

pressure coefficients are obtained from the local velocity vector.

The theory of the panel method is based on the potential equation for subsonic, transonic and supersonic flow

$$\left(1-M_\infty^2\right)\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2}=\frac{M_\infty^2(\kappa+1)}{u_\infty}\frac{\partial\varphi}{\partial x}\frac{\partial^2\varphi}{\partial x^2}$$

which can be derived from the Euler-equation [6]. In this equation M_∞ characterizes the free-stream Mach number and u_∞ the free stream velocity. φ represents the perturbation velocity potential. The isentropic exponent is given by κ. For non-transonic flows, i.e. for subsonic and supersonic flows, this equation can be linearized in the following form

$$\gamma^2\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2}=0 \qquad (2)$$

$$\gamma^2=\left(1-M_\infty{}^2\right)$$

This equation, called Prandtl-Glauert-equation, is linear. This property leads to a more simple mathematical solution compared with that of the Euler- or Navier-Stokes equation. Note that the eq. (2) is elliptic for subsonic free-stream Mach-numbers and hyperbolic for supersonic free-stream Mach-numbers.

When using the panel method one has to make some preliminary assumptions for a correct solution:

- high Reynolds numbers,
- small perturbation, i.e. slender bodies with small angles of yaw and small angles of attack,
- real subsonic or supersonic flow,
- isentropic flow, i.e. only little increase of entropy as the result of weak shock waves in supersonic flow can be admitted.

After a short transformation the Prandtl-Glauert-equation turns in the case of subsonic flow to Laplaces's equation:

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2}=0 \qquad (3)$$

For supersonic flow a similar equation, the wave equation can be defined:

$$\frac{\partial^2\varphi}{\partial x^2}-\frac{\partial^2\varphi}{\partial y^2}-\frac{\partial^2\varphi}{\partial z^2}=0 \qquad (4)$$

The equations (3) and (4) are independent from the direction of the free upstream flow.

The Laplace's equation and the wave equation can be used for the calculation of the flow around bodies, if the following boundary conditions are fulfilled:

- At infinity the upstream flow must be undisturbed, i.e. $\varphi \to 0$
- The flow vector must be tangential at any point of the body's surface,

  i.e. $\vec{u}\cdot\vec{n}=0$ on $S(x,y,z)=0$ (5)
- The perturbations in supersonic flow should only have effects on the downstream flow.

- The body must not have any lift evoking parts.

The detailed derivation is published by Depolt[7].

With the help of eq. (5) and after introducing Green's theorem in eq. (2) the local velocity vector can be formulated as

$$
\begin{aligned}
w(x_{local}, y_{local}, 0) &= 0 \qquad (6)\\
&= \bar{u}_\infty \cdot \bar{n} + \frac{\frac{1}{2} \cdot q(\bar{x})}{\gamma^2 \cdot \bar{n} \cdot \left[B^{-1}\right] \cdot \bar{n}}\\
&+ \frac{k}{4 \cdot \pi} \cdot \iint_{S_{body}} q(\bar{x}) \cdot \frac{[B] \cdot \bar{r}}{R_\gamma^{\;3}} dS(\bar{x})
\end{aligned}
$$

With this relationship the velocity at every point on the surface can be determined so that there is no difficulty to evaluate the pressure distribution on a body's surface.

The definition of the pressure coefficient can be derived from the energy equation

$$c_p = \frac{2}{\kappa \cdot M_\infty^{\;2}} \cdot \left[\left\{ 1 - \frac{\kappa - 1}{2} M_\infty^{\;2} \left(\frac{|\bar{u}|^2}{u_\infty^{\;2}} - 1 \right) \right\}^{\frac{\kappa}{\kappa-1}} - 1 \right] \quad (7)$$

This equation can be written as the following approximate, quadratic expression for the pressure coefficient

$$c_p = -2 \frac{u}{u_\infty} + M_\infty^{\;2} \frac{u^2}{u_\infty^{\;2}} - \frac{|\bar{u}|^2}{u_\infty^{\;2}} \quad (8)$$

Then the pressure coefficient is determined by aid of the equations (7) and (8).

2.2 Discretisation of the Probe Body and Evaluation of the Pressure Distribution

To apply the panel method for the evaluation of the pressure distribution over the surface of the probe body and with that to evaluate the pressure probe characteristics, one has to discretize the body of the pressure probe into a large number of flat panels. Then the panel theory can be applied to each panel. The discretisation of the body is done by using a geometry preprocessor based on the commercial computer program AUTOCAD 10.0. This tool supports the pre-processing very efficiently, so that complex structures can easily be discretized. A transfer program connects the AUTOCAD output data with the program PANSO, which contains the implemented panel theory. The methodology of discretizing a pressure probe is shown in figure 2.

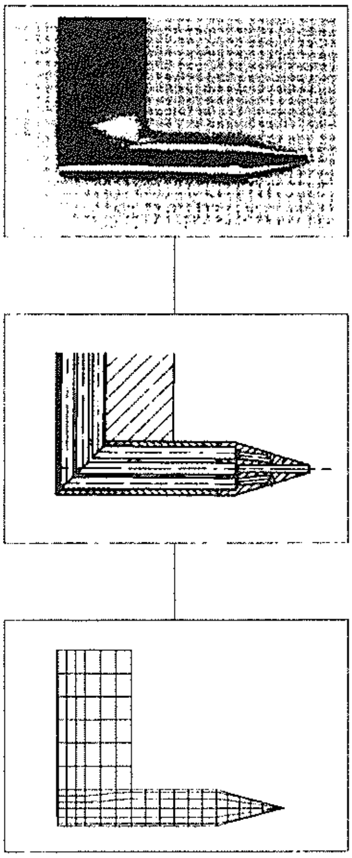

Fig. 2: Methodology of Pressure Probe Discretization

The singularity distribution on the surface can be approximated by constant, linear or square functions. Upon panelwise evaluation of the integral in the eq. (6) the integral equation is converted into a system of algebraic equations for unknown parameters. These unknown parameters represent the strength of the singularity of each panel. By introducing collocation points on the surface of each panel, which are defined by the location of the panels centroids, one can use the above mentioned integral to get a system of linear equations for the strength of source for each panel.

The rear part of the probe head must be assumed as a cone to ensure correct results. This boundary condition is forced by the panel method, which only allows to calculate a cone with an aperture angle smaller than 180 degrees.

The source distribution $q(\bar{x})$ on the surface S_{body} of body-like components is approximated as being constant on each body panel. To determine the unknown parameters one has to use the boundary condition of eq.(6), which requires in the panel's centroid a perpendicular velocity component of zero. The superposition of the velocity with the parallel flow is expressed by the equation

$$\{\bar{u}\} = \bar{u}_\infty + \left[\bar{A}_b\right]\cdot\{Q\} = +\sum_{i=1}^{N} \bar{a}_{bi}\cdot Q_i \qquad (9)$$

where: $Q_i,\ i = 1(1)N$ are the unknown body sources

$\bar{a}^{*}_{bi},\ i = 1(1)N$ are the influence coefficients, which represent the contribution of the singularity parameters to the induced velocity

N is the number of panels

The detailed derivation can be found in Hoeijmakers[6].

First of all the solution vector Q must be calculated in order to determine the velocity vector. After inserting the known velocity vector in eq. (7) and (8), the desired pressure coefficient c_p is obtained.

3. Characteristics of Multi-Hole Pressure Probes

As the first step for the determination of changing probe characteristics in different measurement media the probe characteristic in a undisturbed free jet shall be discussed. Therefore several pressure probe types have been discretized and investigated. A good judgement of the quality of the pressure probe design can be reached by the evaluation of the overall pressure distribution on the probe's surface.

3.1 Basic Definitions

Significant information about the probe characteristics can be obtained by using non-dimensional probe coefficients. The attention shall be focused on two coefficients, which represent the horizontal and vertical angle characteristics. The definitions of the flow vector and the nomenclature of the pressure holes are shown in figure 3.

For the discussion of the pressure probe characteristics the following probe coefficients can be deduced, which are based on the pressures in the holes 0 to 4:

for the yaw angle

$$K_\alpha = \frac{p_3 - p_1}{\Delta p}$$

for the angle of attack

$$K_\beta = \frac{p_4 - p_2}{\Delta p}$$

with

$$\Delta p = p_0 - \frac{p_1 + p_3}{2}$$

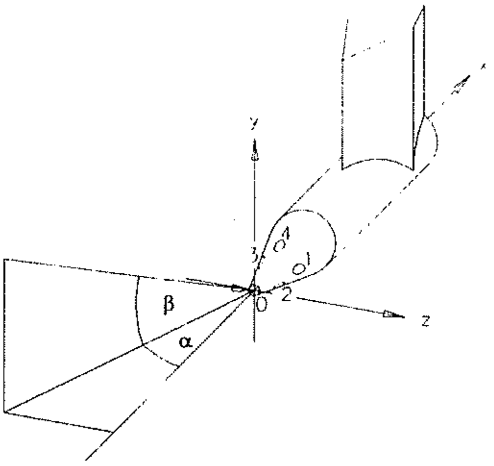

Fig. 3: Nomenclature of Pressure Holes and Flow Angles

The detailed description of the calibration procedure and the use of the non-dimensional probe coefficients can be taken from Koschel/Pretzsch[8]. The numerical evaluation is described as well by Bohn[9].

3.2 Characteristics of Various Pressure Probe Types

In this chapter the discretisation and the evaluation of characteristics of various pressure probe types are presented. The numerical results are compared with experimental data. All experimental and numerical data refer to the Mach number M=0.3.

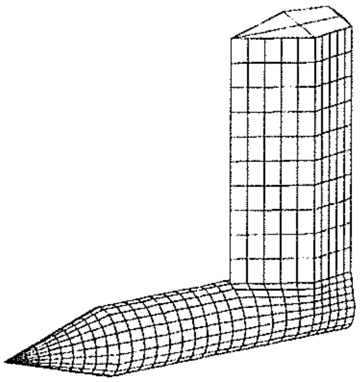

Fig. 4 Discretized Conical Five-Hole Pressure Probe (Type 935)

3.2.1 Conical Five-Hole Pressure Probe

This pressure probe has already been presented by Koschel/Pretzsch[8] at the 9th Symposium in Oxford. This probe type (fig. 4) is especially built for use in transonic turbine stages. The experimental results have been generated from data which have been recorded during tests at the free jet calibration facility of the institute.

The spatial characteristics of this probe have been calculated by the panel code PANSO. The results from this studies can be drawn from the publication of the last year[7]. The general behaviour of the probe due to interference effects of the probe head and the shaft has been already discussed in details.

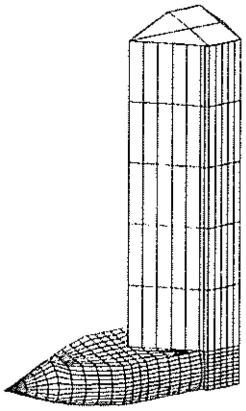

Fig. 5: Discretized Conical Three-Hole Pressure Probe (Type 931)

3.2.2 Conical Three-Hole Pressure Probe

This pressure probe (fig. 5) which was already presented at the 10th Symposium in Brussels[10] has been developed at our institute for the use in high loaded transonic turbine stages with small blade heights. Due to the small channel heights the radial component of the flow vector can be neglected without making a substantial fault.

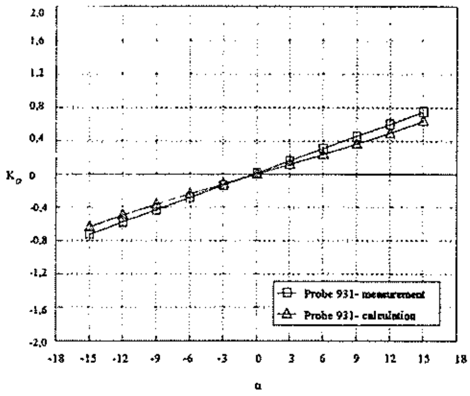

Fig. 6: Comparison between Measured and Calculated Coefficients

This probe is a derivation of the above mentioned five-hole probe. It has a similar shaft and a similar probe head but with removed upper and lower holes. This causes a reduced size and therefore a reduced influence of the probe onto the flow field. The missing upper pressure hole allows to shorten the probe head so that a very compact design can be realised.

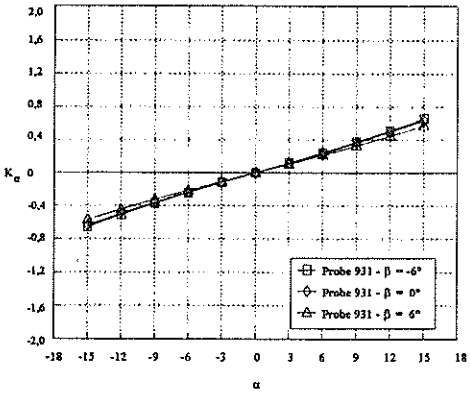

Fig. 7: Influence of the Angle β onto the Probe Characteristics

Figure 6 shows a good conformity between measured and calculated probe coefficients. The influence of the probe characteristics versus different angles of attack are cross plotted in figure 7. It is evident that this probe type is not very sensitive against a changing vertical component of the flow vector.

3.2.3 Three-Hole Pressure Probe NASA-Type

The three-hole pressure probe (fig. 8) has been published by Glawe[11]. It is a combined temperature-pressure-probe to determine the flow conditions at the location of the probe.

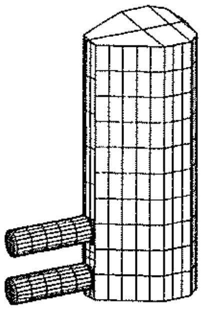

Fig. 8: Discretized Three-Hole Pressure Probe (NASA-Type)

Two pressure holes are placed perpendicular to the surface in the probe shaft for the flow angle determination. The total pressure is recorded by the upper tube of the probe. The lower tube contains a thermocouple for the temperature measurement.

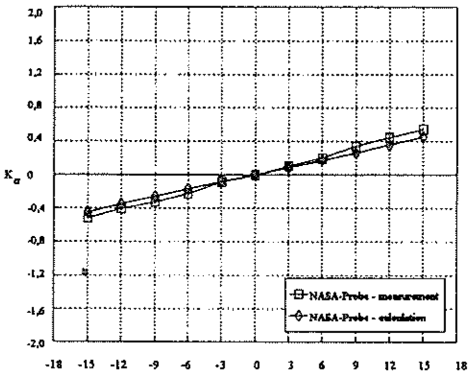

Fig. 9: Comparison between Measured and Calculated Data

Investigations with flow vector variations have shown that this probe is not sensitive refering to a changing vertical component. This behaviour is obvious as the pressure hole cannot be influenced by the shaft.

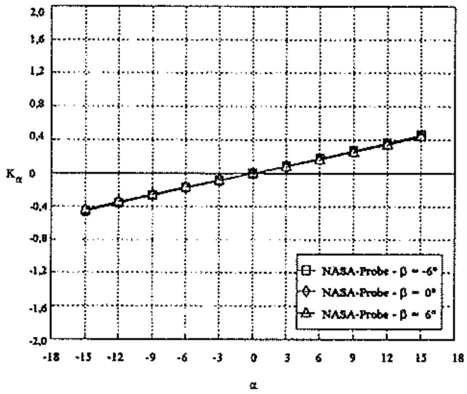

Fig. 10: Influence of the Angle β onto the Probe Characteristics

3.2.4 Long-Head Three-Hole Pressure Probe

This three hole pressure probe (fig.11) is different from the arrangement of the above mentioned pressure probes. The two pressures for the determination of the horizontal flow angle and the total pressure are measured by three tubes which are arranged vertically one-upon-another. Two tubes are sharpened by 45° which leads to the measurement of the static pressure and a component of the dynamic pressure. This pressure hole arrangement is unfavourable as secondary flow effects in the pressure holes can be expected[12].

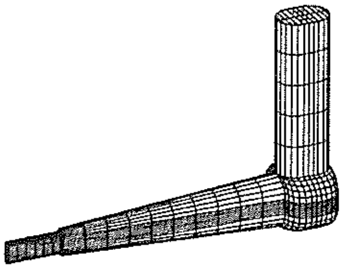

Fig. 11: Discretized Three-Hole Probe (Long-Head Type)

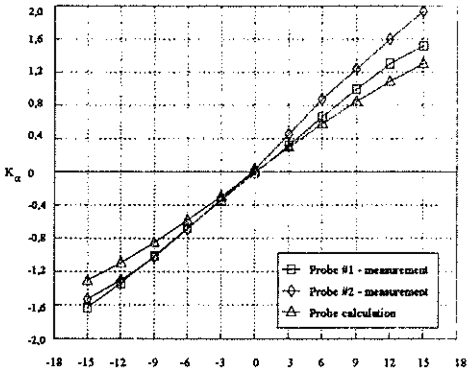

Fig. 12: Comparison between Measured and Calculated coefficients

Variations in the geometry of the probe are normal due to the sharpened tubes. This causes variations in the probe characteristics as it can be seen in figure 12. The non-dimensional coefficients of two probes are compared with the calculation. With respect to the great variations in the geometry, the results of the calculation reproduce the probe characteristics very well (fig. 12).

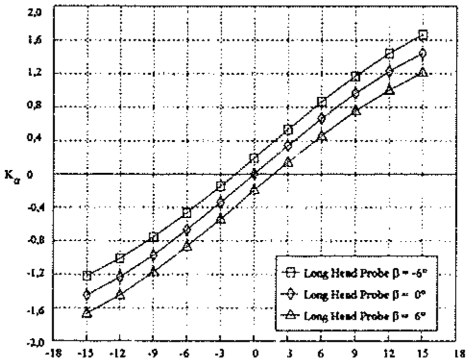

Fig. 13: Influence of the angle β onto the Probe Characteristics

The influence of a non-axial vertical flow is cross plotted in figure 13. Due to the long head, the influence of the shaft onto the pressure holes can be neglected This behaviour can be taken from the curve β=0°.

3.3 Influence of the Flow Characteristics onto the Pressure Probe

The pressure probes used in cascades or turbine stages are usually calibrated either in a free jet or a closed transonic channel depending on the Mach number regime which shall be realised. The pressure probes are introduced after calibration in a test bench under different boundary conditions (fig.14). In small cascades or turbine stages the probe causes a blockage effect. This effect can not be calibrated so far. In order to avoid a misinterpretation of measured flow data due to this interaction the probe should be calibrated under similar conditions.

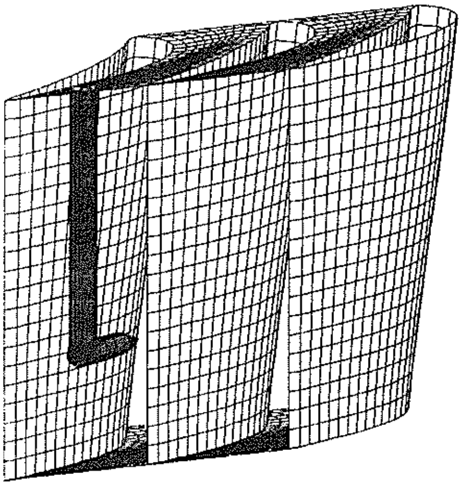

Fig. 14: Pressure Probe Introduced in a Turbine Stage

To make a step forward to solve this problem the changes of the probe characteristics between free-jet and closed-channel calibrations have been investigated. The calculation of our 80x80 mm² calibration channel (fig. 15) shows that no significant difference between the free-jet and the closed channel can be recognized. This results can be confirmed by measurements. The difference of the measured probe characteristics have been determined as minor than 3% which is within the measurement accuracy.

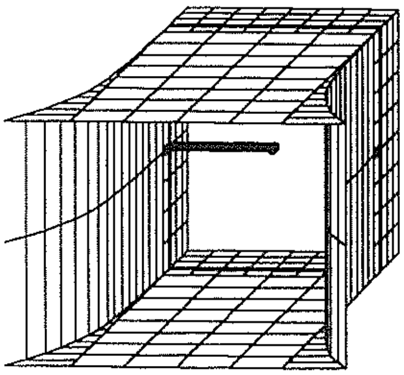

Fig. 15: Pressure Probe Introduced in a Calibration Channel

To evoke interaction effects between the pressure probe and the side walls theoretical investigations using the panel method were made. Figure 16 shows that the size of the channel cross section was reduced down to 4x4 mm². The size of the five-hole pressure probe was held constant during this parameter study. The pressure levels of the holes indicate that the flow is accelerated with increasing blockage up to a maximum. This tendency could not yet be varified by experiments.

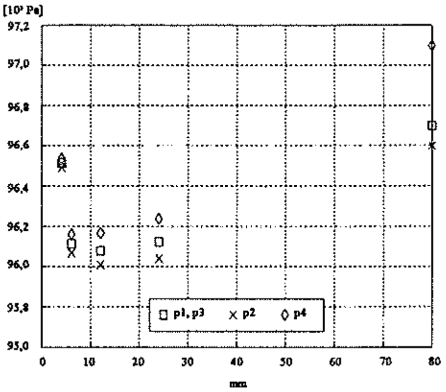

Fig. 16: Influence of Channel Cross Section onto the Probe Characteristic

With these results one can state that the panel code PANSO in connection with the geometry preprocessor has the capability to evaluate these flow effects.

4. Conclusion

The problems of the application of the pressure probe measuring technique are well known. They are caused by the fact that the probes usually can not be calibrated under the same conditions as they are exposed during later use. During the calibration in a free-jet or a closed channel side wall effects can be neglected. Using the pressure probes in a cascade or a turbine stage the measurement cross section can be quite different, usually smaller. A numerical approach based on the panel method has been developed which supports a theoretical treatment of this problem in a wide range of compressible flow conditions.

In a first step the panel method was used to determine the free-jet probe characteristics. The results, which are described in this paper, show that the characterstics of three-hole pressure probes can be reproduced theoretically very well. Therefore the panel method is a very useful design tool which supports the construction process of pressure probes.

With this technology a new perspective for the investigations of pressure probe development is given. The effects of changing geometry parameters can easily be studied by the results of the calculations. The major points of interest are the consequences of changes in the probe head-shaft-region. Systematic investigations in different probe head-probe shaft distances, different arrangements of the pressure holes and different aperture angles of conical probe heads have already been done and has partially been presented[7]. The great advantage of this theoretical procedure versus parameter studies by probe models is the time and cost efficiency.

The results of the calculations under closed channel conditions are very helpful to simulate flow effects and to recognize tendencies. A high precision of the calculation can not be expected due to the simplifications which were made. No considerations of the boundary layer or friction effects are implemented yet. This shall be the task of the future work.

5. References

[1] Ballmann, J. (ed.) : "Panel method in fluid mechanics with emphasis on aerodynamics", Vieweg, Braunschweig, 1988

[2] Woodward, F.A. : "An improved method for the aerodynamic analysis of wing-body-tail configuration in subsonic and supersonic flow", NASA CR 2228, 1973

[3] Carmichael, R.L.; Erickson, L.L. : "PAN-AIR - A higher order panel method for predicting subsonic or supersonic linear potential flows about arbitrary configurations", AIAA Paper 81-1255, 1981

[4] Magnus, A.E.; Epton, M.A. : PAN AIR - A Computer Program For Predicting Subsonic or Supersonic Linear Potential Flows About Arbitrary Configuration Using a Higher Order Panel Method, Vol.1 Theory Document (Version 1.0)", NASA-CR 3251, 1980

[5] Hoeijmakers, H.W.M. : "Panel Method for the Determination of the Aerodynamic Characteristics of Complex Configurations in Linearized Subsonic and Supersonic Flow", Report NLR TR 80124 U, National Aerospace Laboratory, The Nederlands, 1980

[6] Hoeijmakers, H.W.M. : "Panel Methods in aerodynamics - Some Highlights", Extract from Report NLR TR 80124 U

[7] Depolt, Th.; Koschel, W. : "Investigation on Optimizing the Design Process of Multi-Hole Pressure Probes for Transonic Flow with Panel Methods", Presented at 'International Congress on Instrumentation in Aerospace Simulation Facilities (ICIASF) '91', Rockville, MD., Oct. 27-31, 1991, IEEE Publ. 91CH3028-8

[8] Koschel, W.; Pretzsch, P. : "Development and Investigation of Cone-Type Five-hole Probe for Small Gas Turbines", Presented at '9th Symposium on Measuring Techniques for Transonic and Supersonic Flows in Cascades and Turbomachines', Oxford, 1988

[9] Bohn, D.; Simon, H. : "Mehrparametrige Approximation der Eichräume und Eichflächen von Unterschall- bzw. Überschall-5-Loch-Sonden", ATM, März 1975, Lieferung 470

[10] Depolt, Th.; Vinnemeier, F.; Koschel, W. : "Investigations on Minimizing Blockage Effects of Multi-Hole Pressure Probes in Transonic Turbine Stages", Presented at '10th Symposium on Measuring Techniques for Transonic and Supersonic Flows in Cascades and Turbomachines', Brussels, 1990

[11] Glawe, G.E.; Krause, L.N.; Dudzinski, T.J.: "A Small combination Sensing Probe For Measurement of Temperature, Pressure and Flow Direction", NASA TN D-4816

[12] Fransson, T.; Sari, O. : "Characteristics of Aerodynamic Five-Hole-Probes in Transonic and Supersonic Flow Regimes"; Presented at '6th Symposium of Measuring Techniques in Transonic and Supersonic Flows in Cascades and Turbomachines', Ecully, 1981