Calibration, Modeling and Data Evaluation Procedures for Aerodynamic Multihole Pressure Probe Measurements on the Example of a Four Hole Probe

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This paper describes the calibration and data evaluation methods used at the ETH Turbomachinery Laboratory for aerodynamic measurements with fast response pressure probes.

The static aerodynamic calibration test facilities, test instrumentation and calibration procedures used are briefly presented. The choice criteria for adequate calibration coefficients are discussed for the example of a cylindrical four hole probe.

The mathematical modeling of the probe behaviour in the angular range required by many applications seems to be critically dependent on the data evaluation method used. For this reason, several ways of modeling calibration coefficients - based on polynomial models and on look-up tables - are compared in terms of accuracy and computation time.

1. Introduction

Flow field measurements with multihole pressure probes require an accurate probe calibration, adequate calibration coefficients and modeling of those coefficients as well as an effective data evaluation method to treat the very large amount of data collected during fast response measurements.

The cylindrical, fast response pressure probe design with 4 miniature pressure transducers (named as type Z4LS) is used to illustrate the concepts described in the following sections.

2. Calibration of multihole pressure probes

2.1 Calibration steps

The calibration process of sensor equipped fast response probes can be divided in three main steps. Conventional pneumatic probes only require step c.

a. Electrical calibration of the sensors

The sensors packaged in completed pressure probes are calibrated as function of excitation voltage and output signal in an environmental chamber at a temperature and a pressure range matching the application conditions. Due to the long term drift of the sensors, this step should be repeated from time to time.

b. Frequency response calibration of the pressure sensors

The dynamic response of pressure probes is dependent on the type of sensors used and the chosen packaging configuration (sensor chip flush mounted or recessed in a tapping cavity). Experiments have been carried out in a shock tube e.g. by GOSSWEILER, HUMM, KUPFERSCHMIED //1// and AINSWORTH, ALLEN //2// to calibrate the sensors dynamically and to obtain the transfer function (gain and phase). The dynamic response of the sensors is not affected in a bandwidth of typically 40 kHz. This step is necessary only for new sensor types or if important changes in the packaging configuration are considered.

c. Aerodynamic calibration of the probes

It has been widely recognised that static calibration data applied to dynamic flow measurements can lead to significant errors. However, since the correction model of dynamic effects proposed by HUMM, VERDEGAAL //3// is based on static calibration data, careful performed aerodynamic calibrations under static conditions will still be required in the case of unsteady applications.

2.2 Static aerodynamic calibration facilities

At the Turbomachinery Laboratory, a supersonic and a subsonic facility have been built for aerodynamic probe calibration purposes. Both tunnels as well as a pipe flow experiment designed for comparative flow measurements (GOSSWEILER, HERTER, KUPFERSCHMIED //4//) are connected to the wind tunnel air supply, which consists of a radial compressor with DC drive. Special attention has been paid to dust filtering and temperature control to carry out hot-wire measurements.

A Laval tunnel with a nozzle cross section of 200 x 40 mm permits probe experiments up to Mach 1.4. Pneumatic and optical methods such as holography have been used to investigate the flow field around probe bodies.

The axisymmetric nozzle facility (exit diameter 100 mm) shown in fig. 2.2.1 is dedicated to probe calibrations up to Mach 0.9. Fairly steady stagnation conditions, uniform velocity profiles and a flow turbulence level below 0.3 % could be achieved with this constant flow type tunnel. The flow temperature is controlled within ± 0.05 °C by means of an air/water heat exchanger to perform satisfactory calibrations of sensor equipped probes. The facility can be operated in the temperature range between 10°C and 50°C. The probes are positioned in the jet with a multiaxis actuator system sketched in fig. 2.2.2. The yaw and pitch angle rotations are remote controlled. The positioning precision lies within ± 0.05° for both yaw and pitch angles.

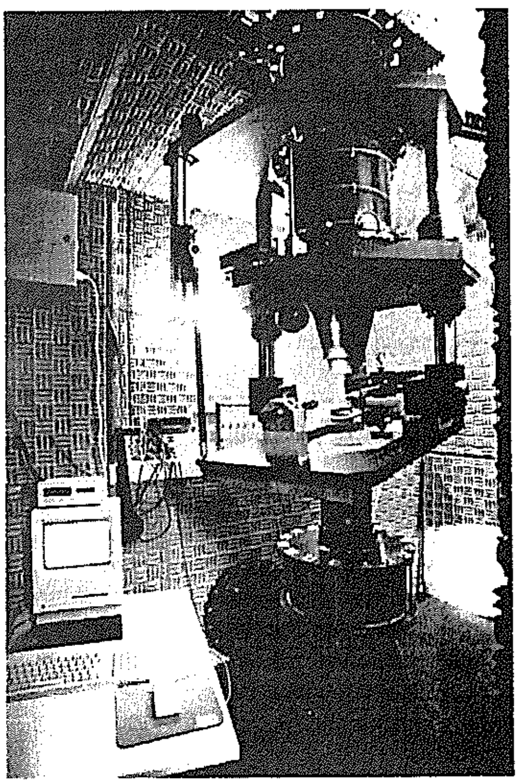

Fig. 2.2.1 *Jet nozzle calibration facility*

To enhance both calibration quality and point density on one hand and to avoid human exposure to the extremely loud jet environment on the other, the aerodynamic calibration process is fully automatic. A Macintosh computer controls the probe actuators according to an input spreadsheet file and acquires data on flow conditions in the test section and the tapping signals at the corresponding probe position. In the case of conventional pneumatic probes, tapping pressures are measured with a multichannel pressure scanner. A "Paroscientific" pressure transducer system is used as a reference for the calibration of the scanner, which is equipped with "Sensym" transducers.

An algorithm checks the pressure levels measured at the different probe tappings to avoid any transient effects. As soon as the collected data are statistically representative, they are stored on computer and the actuators turn the probe to the next position. During the calibration process, the offsets of the scanner sensors are readjusted automatically from time to time. With a conventional four hole probe, the system performs 100 to 300 calibration positions per hour depending on the jet Mach number and the requested angular range.

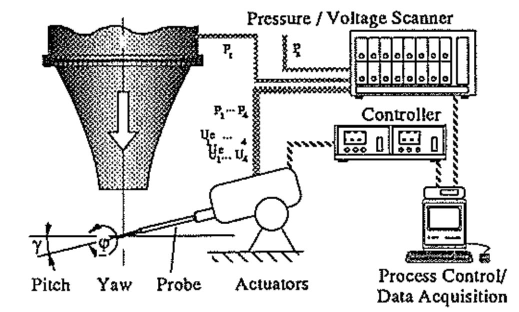

Fig. 2.2.2 *Calibration set-up*

Due to the lower stability of packaged miniature sensors, the aerodynamic calibration of fast response probes should be as short as possible. For that purpose, the computer drives a high precision multichannel voltmeter. A calibration rate of about 600 points per hour seems to be within reach.

Another way to proceed is to use calibration data of a conventional pneumatical probe of similar geometry and to correct small discrepancies by using a few discrete calibration points of the fast response probe version.

2.3 Typical Calibration Results of Probe Z4LS

Figure 2.3.3 shows four non-dimensional pressure coefficients $C_p = p / \left(\frac{\rho}{2} C_{ref}^2\right)$ of the Z4LS probe (fig. 2.3.1) in the jet nozzle as function of yaw and pitch angles at Mach 0.2. The tappings are numbered according to the spherical probe coordinate system of fig. 2.3.1 (with φ for yaw and γ for pitch). The mesh increment corresponds to the calibration grid (2° steps with yaw $[-30° \leq \varphi \leq 30°]$ and pitch $[-20° \leq \gamma \leq 24°]$).

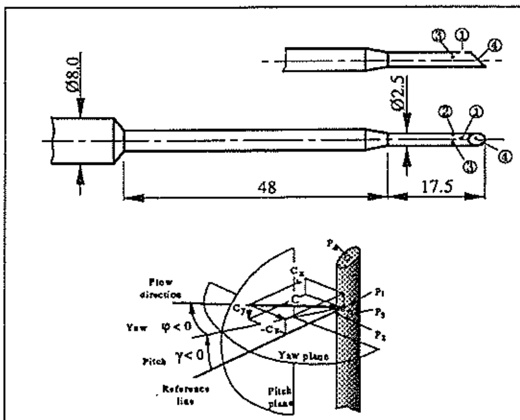

Fig. 2.3.1 *Cyl. pressure probe Z4LS (Ø 2.5 mm) with spherical probe coordinate system*

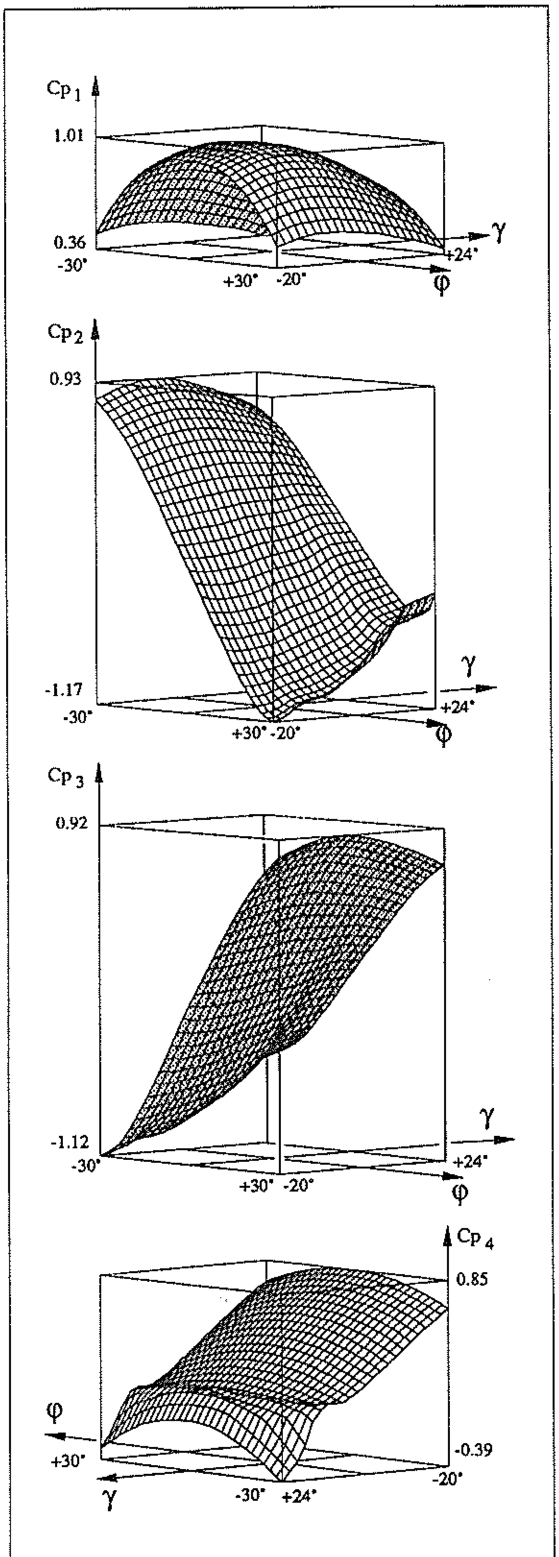

Fig. 2.3.3 *C_p coefficients of probe Z4LS (Ma = 0.2) (lower surface shadowed, see C_{p1} and C_{p3})*

3. Aerodynamic calibration coefficients

3.1 Choice criteria for coefficients

In the following, incompressible air flow conditions are assumed.

The above mentioned pressure coefficient C_p for each hole has to be transformed into a set of non-dimensional coefficients to allow further evaluations of the flow quantities. Basically, the independent flow quantities that can be measured with a four hole probe are both flow angles φ and γ and both total and static pressure. In addition, the temperature information can be gained from the sensor's Wheatstone bridge signals of fast response probes (GOSSWEILER, HUMM, KUPFERSCHMIED //7//).

What are the criteria to chose such coefficients? Since there are many different coefficient definitions, the choice is merely dictated by the probe type and the availability of a reference pressure. The aerodynamics is dependent on the probe geometry and thus influences directly the shape of the C_p surfaces. Estimations on the expected application such as angular ranges and Mach number are important factors as well.

Regarding the following data evaluation procedures, the variables should be as independent as possible from each other. For instance the coefficient K_φ plotted in fig. 3.2.1 is yaw sensitive and depends weakly on the pitch. This facilitates the data processing.

3.2 Typical calibration coefficients

The following coefficient definition applied to the Z4LS probe geometry and to a restricted angular field fulfils the above criteria and shows satisfactory results:

- Angular sensitivity (fig. 3.2.1):

$$K_\varphi(\varphi,\gamma) = \frac{p_2 - p_3}{p_1 - \frac{p_2+p_3}{2}}, \quad K_\gamma(\varphi,\gamma) = \frac{p_1 - p_4}{p_1 - \frac{p_2+p_3}{2}}$$

- Pressure sensitivity (fig. 3.2.2):

$$K_t(\varphi,\gamma) = \frac{p_t - p_1}{p_1 - \frac{p_2+p_3}{2}}, \quad K_s(\varphi,\gamma) = \frac{p_t - p_s}{p_1 - \frac{p_2+p_3}{2}}$$

Another, but important choice criteria is the continuity of the definition domain. Mathematical singularities or ambiguities can render further data evaluation impossible. The plots of the coefficients in fig. 3.2.1 illustrate the high gradients encountered at the angular extremities of calibration ranges. For this reason, the above defined coefficients are limited to an angular range between about ± 32° in both yaw and pitch variation. Around ± 35°, the functions present a pole due to the denominator tending towards zero.

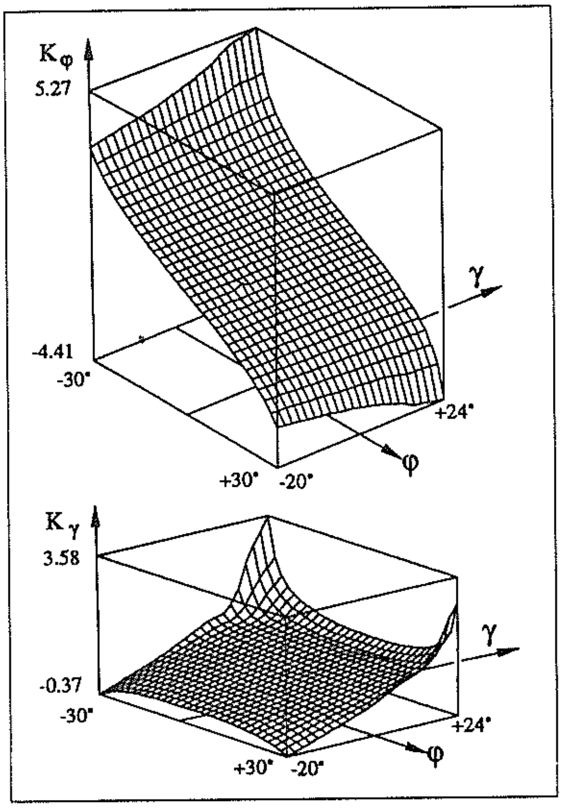

Fig. 3.2.1 *Angular sensitive coefficients (Ma = 0.2)*

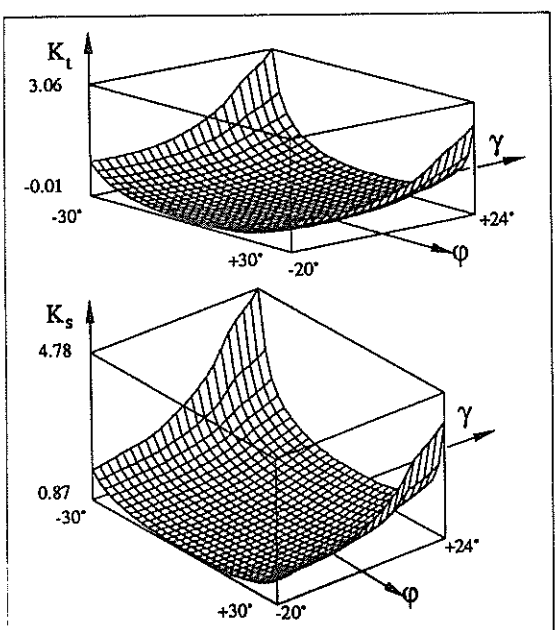

Fig. 3.2.2 *Coefficients for p_t and p_s (Ma = 0.2)*

4. Aerodynamic calibration data modeling

Discretely collected calibration information has to be modelled to allow an effective numerical data evaluation of later measurements. This mathematical description of the probe characteristics can be achieved in two different ways:

a. Non-parametric models

With non-parametric models, the calibration data are stored in look-up tables: the data are in raw form as coordinates x, y and z of the coefficient surfaces K. Actually, this is not a model because no data has been transformed or approximated, except perhaps a "special conditioning" of outlying points. Further evaluation can be carried out with local interpolation based on bivariate functions available as optimised routines in mathematics libraries on mainframe computers (generally restricted to equidistant mesh point distributions). Obviously, the accuracy is strongly dependent on the mesh density.

b. Parametric models

Calibration data are fitted with mathematical model functions. Polynomials in two variables seem to be adequate for our surface shapes (at least within normal angular ranges) and have been presented in several papers, for instance by BOHN, SIMON //6//:

$$z = f(x,y) = \sum_{i=0}^{m} \sum_{j=0}^{n} k_{ij} x^i y^j ,$$

m and n being the polynomial degree in x and y direction. Based on Lagrange's least squares method described e.g. by LANCASTER, SALKAUSKAS //7//, the polynomial coefficients k_{ij} are determined by solving the "normal equations".

The choice of the degree of the polynomial has to be carefully balanced between the modeling accuracy and the resulting number of coefficients $n_k = (m+1)(n+1)$ (computation time). The degree of freedom n_f of the model, defined as the difference $n_f = n_p - n_k$, is a rough indicator for the statistical precision of the fitted parameters of the model (n_p: number of calibrated points).

Either a direct or an indirect flow angle evaluation method is suitable for both model types (section 5.2)

5. Flow data evaluation

5.1 From probe signals to flow quantities

The steps required to convert probe measurement data into flow quantities are described in fig. 5.1.1. Indicated quantities present a circumflex mark.

Once the sensor voltages have been converted into pressure and temperature signals (Model Based Reconstruction //5//), both flow angles are determined with the coefficients $\widehat{K\varphi}$ and $\widehat{K\gamma}$ using either an indirect or a direct method. The total pressure and then the static pressure are calculated by using the flow angles $\hat{\varphi}$ and $\hat{\gamma}$ (here the angular accuracy is important to avoid severe errors on pressure information). Other quantities such as flow temperature and velocity are computed later on. In a last step, the results are transformed into the mean stream coordinate system.

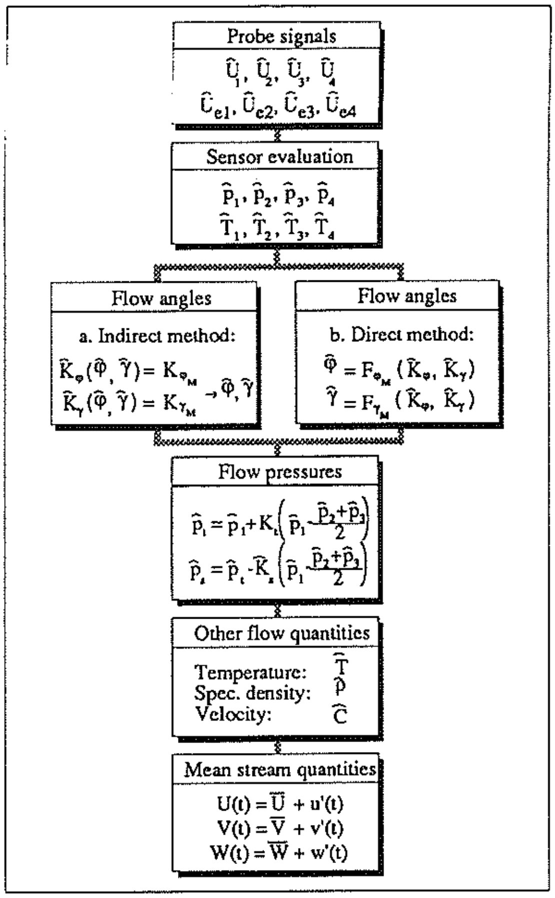

Fig. 5.1.1 *Measurement data evaluation steps*

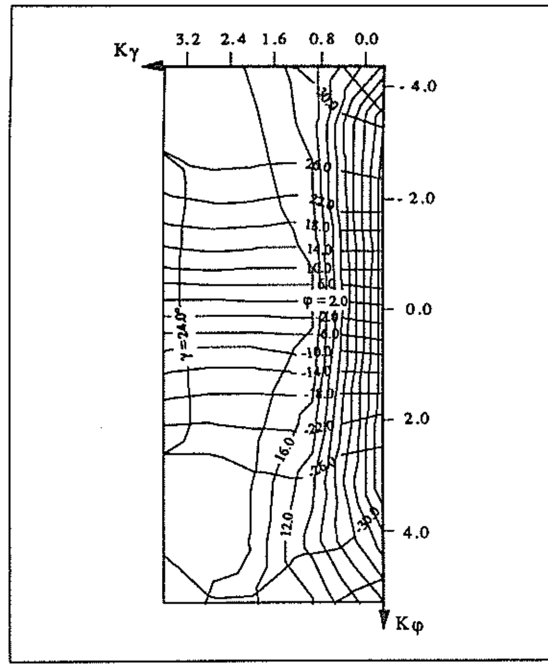

Fig. 5.2.2 *Direct solving of* $\varphi, \gamma = F(K_\varphi, K_\gamma)$

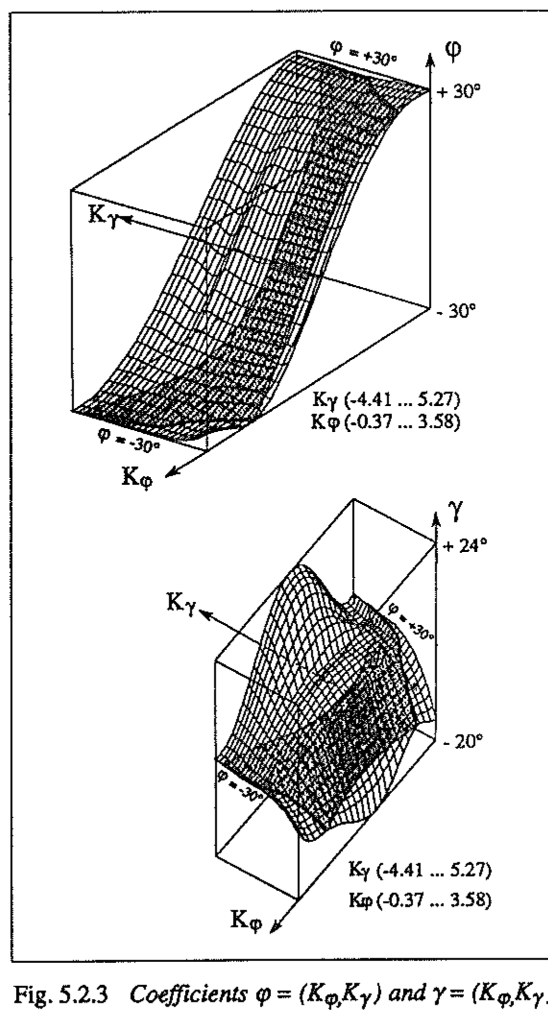

Fig. 5.2.3 *Coefficients* $\varphi = (K_\varphi, K_\gamma)$ *and* $\gamma = (K_\varphi, K_\gamma)$

5.2 Angle solving methods

Since the accurate knowledge of the flow angle is the key to the evaluation of further flow quantities (such as total and static pressure and velocity), great attention should be paid to this step, which can be solved with two different approaches:

a. "Indirect" method

$K\gamma_M$ and $K\varphi_M$ are plotted as function of yaw and pitch angles (fig. 3.2.1) with a grid corresponding to the calibration mesh. The angle solution $\hat{\varphi}$ and $\hat{\gamma}$ is found by iterations: each of the two values $\hat{K}\gamma$ and $\hat{K}\varphi$ calculated with the indicated probe pressures represents a contour line in one of the plots in fig. 3.2.1. If both plots are superposed, the intersection point of the contours gives the angle solution. This well known method has been used e.g. by HENEKA //8//. A Newton iteration algorithm has been implemented in our evaluation software to test this method.

b. "Direct" method

If the dependent and the independent variables of the plots 3.2.1 are exchanged to

$$\varphi_M = F_{\varphi_M}(K_\varphi, K_\gamma)\,;\ \gamma_M = F_{\gamma_M}(K_\varphi, K_\gamma)\ ,$$

each yaw or pitch angle appears to be dependent only on the coefficients K_φ and K_γ (fig. 5.2.2, 5.2.3).

The flow angles can then be calculated directly from these equations and do not require any iteration. An important fact to note is that the dependent variables are no more equidistant: some areas having a low density of points, fitting parametric models can lead to systematic errors in these regions or even risk to oscillate between data points.

In the case of look-up tables, interpolations between distant points can affect the accuracy. As already mentioned, most of the bivariate interpolation procedures available in mathematics software libraries are restricted to equidistant grid points. Since the calibration data of the direct angle evaluation method presents a scattered distribution, the interpolation could be performed by triangulations (SPÄTH //9//) at costs of an important increase of computation time. Therefore, it seems easier to transform the scattered distribution of the calibration data points into a finer equidistant mesh by means of a bivariate spline interpolation procedure.

Depending on the point density in the scattered representation, such procedures involving splines of higher degrees can lead to considerable inaccuracies due to oscillations. Improvements have been observed by interpolating the calibration data in the original representation $K_\varphi = f_\varphi(\varphi,\gamma)$ and $K_\gamma = f_\gamma(\varphi,\gamma)$. With regards to calibration quality, this can obviously be done only to a certain extent!

6. Comparison of evaluation methods

The 3 mentioned calibration data options (indirect method with polynomial modeling / direct method with look-up tables / direct method with polynomial modeling) have been implemented in the evaluation software. The following sections describe the investigations undertaken to quantify the modeling accuracy of the direct evaluation with polynomial modeling. A comparison of the computation time required for the three options is also presented.

6.1 Angle modeling and evaluation accuracy

To get an estimation of the accuracy of parametric coefficient modeling (direct method), the previous surfaces $\varphi = f(K_\varphi, K_\gamma)$ and $\gamma = f(K_\varphi, K_\gamma)$ have been fitted with two dimensional polynomials of degree varying from 3 to 9. As an example, three cases with different angular ranges have been treated and the ranges taken into account in the models have been varied according to table 6.1.1. Case III has the smallest ranges to enhance local "goodness-of-fit" in the vicinity of small mean flow angles ($\varphi \approx 0°$, $\gamma \approx 0°$). The standard deviation between the solution and the model, defined e.g. for yaw as

$$s_\varphi = \sqrt{\frac{1}{n_p-1}\sum_{i=1}^{n_p}\left(\hat{\varphi}_i-\varphi_i\right)^2}$$

are plotted in fig. 6.1.2 for all cases.

The conditioning of the normal equation matrix sets a limit of the polynomial degree as 7 for case I.

As expected from a cylindrical probe, the pitch angle is more difficult to fit and the errors do not decrease perceptibly with higher polynomial degrees. Obviously, the accuracy of modeling can be enhanced by reducing both angular ranges, but remains strongly dependent on the application.

Generally, the probes are aligned with respect to the mean values of yaw and pitch. Typical distributions of the probability of the flow vector to be in a certain angular range have shown in the case of turbulent flows that the accuracy in the central area around $\varphi \approx 0°$ and $\gamma \approx 0°$ should be as high as possible and that the outer area is only of secondary importance. Case III fulfils these requirements in the range of ±20° for yaw and ±24° for pitch. The evaluation of angles lying out of these ranges is carried out either with case II or case I, depending on the flow angle distribution.

Case	Yaw [°]	Pitch [°]
I	-30 .. 30	-20 .. 24
II	-26 .. 26	-18 .. 18
III	-20 .. 20	-14 .. 14

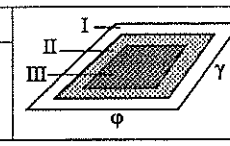

Table 6.1.1: *Angular definition of modeling cases*

An interactive program package has been designed for the modeling process. Graphical output capabilities (3D) and residuals statistics (differences between data and model) permit to predict the model behaviour and to calculate the angle evaluation accuracy.

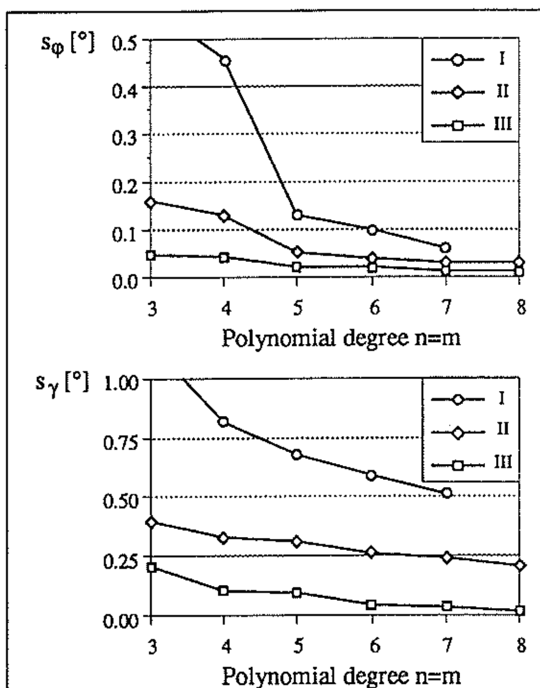

Fig. 6.1.2 *Accuracy of yaw and pitch angle (parametric model, direct evaluation method)*

Accuracy investigations on non-parametric modeling are not completed at this time and results can not be shown here. Obviously, interpolation in look-up tables does not lead to systematic model errors. On the other hand, random errors from calibration may affect considerably the precision depending on the number of data points used for interpolation and on calibration accuracy.

6.2 Accuracy of pressure evaluation

The next figures show the standard deviation of total pressure, static pressure (in % of dynamic head) and flow velocity magnitude C (according to the angular ranges of table 6.1.1) as functions of the polynomial degree of the angular modeling. The models for K_t and K_s used to determine p_t and p_s were sixth degree polynomials.

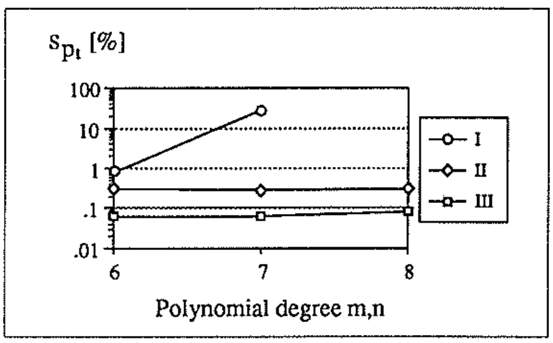

Fig. 6.2.1 Standard deviation of total pressure

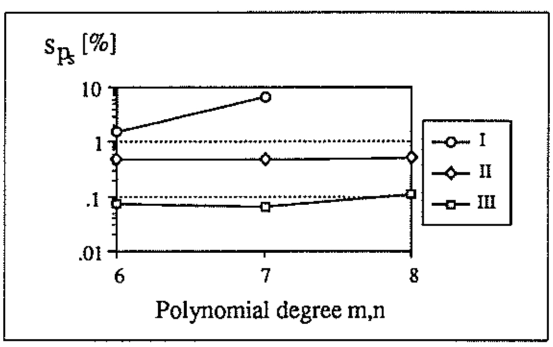

Fig. 6.2.2 *Standard deviation of static pressure*

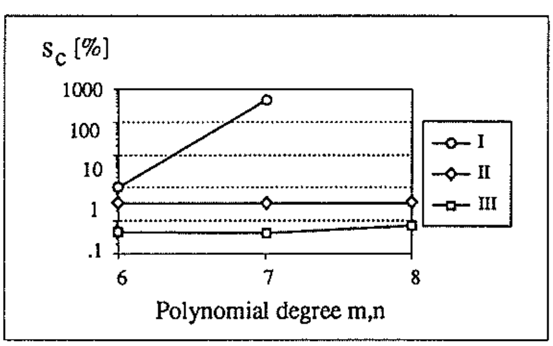

Fig. 6.2.3 *Standard deviation of resulting flow velocity C.*

Any small inaccuracies in the flow angles evaluation results in perceptible errors in pressure. Thus, noticeable inaccuracies of p_t, p_s (and flow velocity) can result. This effect can clearly be seen in fig. 6.2.1, 6.2.2 and 6.2.3 where the standard residuals increase with higher polynomial degree.

6.3 Some considerations on computation time

The most time consuming step in the evaluation procedure is the flow angle computation. Therefore, the 3 options (direct interpolation in look-up tables, direct and iterative solving methods with parametric models) have been implemented in the software program "AW-System" running on a mainframe DEC VAX 9000-420 under VMS. This software has been developed at the Turbomachinery Laboratory especially for the data evaluation of large time series (HERTER, CHRISANDER, GOSSWEILER //10//). As a first example (test), time series of 262'000 points collected in a turbulent pipe flow at Ma = 0.2 (GOSSWEILER, HERTER, KUPFERSCHMIED //4//) have been evaluated. The CPU time (table 6.3.1) was measured from the conversion of sensor signals up to the determination ofall flow quantities (fig. 5.1.1).

Methods and Models	CPU time
Direct solving methods	
Look-up tables (Interpolation)	2:27 min
Parametric model (Polynomials)	2:20 min
Iterative solving methods	
Parametric model (Polynomials)	15:08 min

Table 6.3.1 *Comparison of evaluation time for 262'000 data points (VAX 9000-420)*

Because of the much lower CPU time required, direct solving methods are more attractive for large time series than iterative solving methods. Both parametric models were sixth degree polynomial. The resulting CPU time difference between the two direct solving methods would increase with higher polynomial degree. The time needed by the iterative solving method is of course strongly affected by the required iteration accuracy.

6.4 Detection of outliers (fool points)

To avoid random errors using the look-up table method, it is of prime importance to detect and eliminate outliers remaining from the calibration. Even a single fool point can influence significantly the accuracy of modeling. Usually, such points are detected during the modelisation process by the interactive program with 3D graphical output capabilities (section 6.1). A good modeling without systematic errors shows a Gauss distribution of the residuals. Drawn in a quantile plot (DANIEL, WOOD //11//), such residuals lie on a straight line. Since the residuals of the polynomial model (fig. 6.4.1) do not lie on a straight line, some systematic errors are present. Points far away from this ideal line are random errors. These are suspected to be undesired outliers and are eliminated.

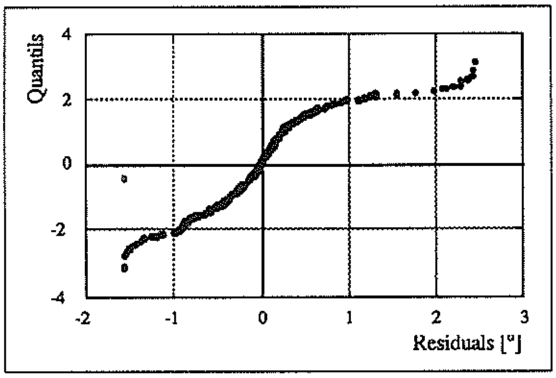

Fig. 6.4.1 *Typical Quantile plot of polynomial modeling of calibration surfaces*

7. Conclusions

To enhance the accuracy of both the calibration and the modeling, the calibration grid should be as narrow as possible. For this purpose, an automatic aerodynamic facility is very accurate and comfortable to use.

Large angular ranges are required in many applications, but an accurate modelisation is difficultly obtained over the whole range. Since the determination of both total and static pressure is very sensitive to the angles as input, great attention should be paid to the angle evaluation. The typical angle distribution of turbulent flows, concentrated around the aligned probe configuration, renders the angular accuracy less critical in the area of large angles.

Due to the lower CPU requirements, direct evaluation methods show a better potential than indirect methods to evaluate large time series as in the case of fast response probe measurements.

8. Acknowledgements

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Abbreviations, List of Symbols

C_p	non-dimensional pressure coefficient	[-]
K_φ, K_γ	angular calibration coefficients	[-]
K_t, K_s	pressure calibration coefficients	[-]
k_{ij}	polynomial coefficient	[-]
Ma	Mach number	[-]
m, n	polynomial degree (x, y-axis)	[-]
n_p	number of calibration points	[-]
n_k	number of coefficients	[-]
n_f	degree of freedom	[-]
p	number of coefficients	[-]
p_i	pressure at tapping i	[mbar]
T_i	temperature of sensor membrane i	[K]
s	standard deviation	
φ, γ	yaw angle, pitch angle	[°]

Indices

t	total
s	static
^	indicated
M	model

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