Session 3 - Hot Wire Measurements

THERMAL ANEMOMETER MEASUREMENTS OF 3D VELOCITY AND TURBULENCE IN TURBOMACHINES - A CRITICAL REVIEW -

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ABSTRACT

The measurement of three-dimensional mean velocity and turbulence components using thermal anemometers is well established in laboratory situations such as low-speed wind tunnels, but the application of these techniques to turbomachines brings problems of high turbulence intensity, a mixture of periodic and random unsteadiness, and probe access which is often less than ideal. This paper reviews probe types and configurations, data acquisition and data processing methods and the special precautions which must be taken. It is argued that the interactions between probe, data acquisition system and processing algorithm are sufficiently strong that the measuring chain must be designed as a system rather than built up from components selected individually. Suitable systems are identified, while others previously proposed are found to be unable to achieve their objectives. The emphasis is on simple robust systems which work on the test bed as well as in the laboratory.

INTRODUCTION

The origin of this project was an attempt to measure the three-dimensional steady velocity and turbulence in a turbomachine using a thermal anemometer. A particular requirement was that the measurements were to be made in an 'industrial' rather than a 'laboratory' situation, so that the equipment had to be simple to operate, reasonably robust, and also implied that access to the test section was limited and unlikely to be optimal. A thermal anemometer was selected in preference to a laser anemometer because of the high cost of the multi-beam or multi-colour laser required for full three-dimensional measurements. Thermal anemometers are usually associated with low speed flows in carefully controlled conditions, and this project was viewed as a pilot study to gain experience in using such devices in true turbomachine conditions.

The characteristics of turbomachine environments as they apply to thermal anemometers can be summarised as follows:

- The mean flow is three-dimensional, but there is often a predominant direction. This can be beneficial, because the sensor response equations often have a unique solution only within a range of flow directions such as an octant of a sphere, and prior knowledge of this is necessary. The knowledge that one velocity component is large in comparison with the other two *may* simplify the data processing, but in practice is unlikely to because . . .
- The turbulence is three-dimensional, not necessarily small and without a predominant direction.
- In separated regions, there is no predominant direction and the turbulence intensity is high. Thermal sensors have difficulty under these circumstances because of the problems mentioned above, and because the inherent non-linearity of the response is likely to introduce errors at high intensity.

- Access can be a problem. It is usual for the probe stem to be normal to the flow, not parallel as in conventional anemometry. This can cause vibration and prong interference problems.

There are wide ranges of probes, data acquisition systems and data processing techniques available for making such measurements. These are all interrelated, and if this is not recognised, a poor system may result which will not perform well and may limit the measurements which can be made. In most of the literature individual systems are described, often at length and in great detail, with little or no attempt to compare different systems or solutions. The emphasis of this paper is on the interaction between the components of the system: the probe, the data acquisition system and the data processing algorithm, in order to show how the characteristics of each influence the overall performance. This information has largely been learnt through experience of constructing and running systems. In one case it was found that a particular combination of components renders it impossible to measure turbulence quantities, and this is contrary to what has previously been implied in the literature.

PROBE DESIGN

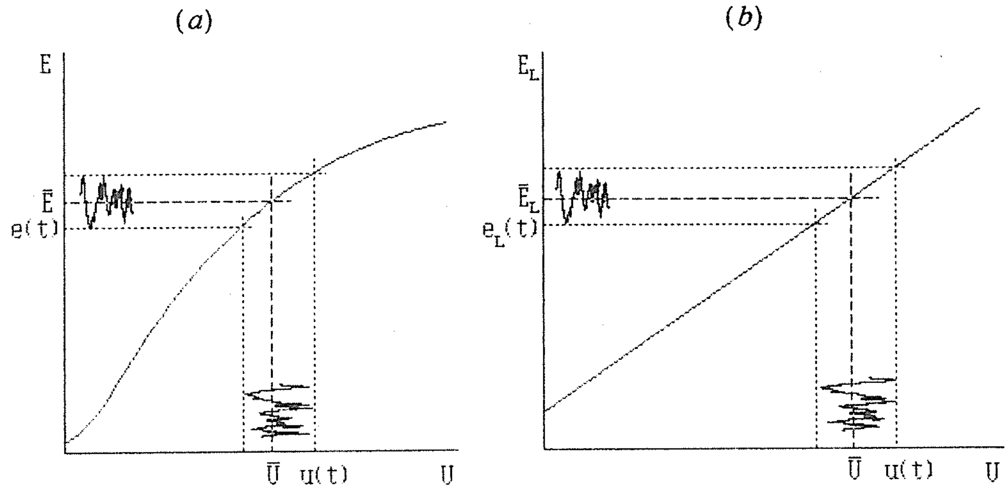
Three-dimensional flow measurements are normally made with a three-sensor probe, or a single-sensor probe rotated about the measurement point. Rotation is only normally feasible about the axis of the probe stem, which in turbomachines is almost always introduced normal to the streamlines (contrary to low-speed wind tunnel experiments, where the probe axis is most commonly in the streamwise direction). This implies that the sensor must be slanted relative to the probe axis. A rotated single-sensor probe and an axisymmetric three-sensor probe can, for data processing purposes, be treated in the same way. Alternatively, three-sensor probes can be non-axisymmetric, although generally they must be specially made.

A special case of slant sensor is one which makes angle of 54.7° to the probe axis. Three such sensors set at 120° to each other in a plane normal to the probe axis (or one sensor rotated to three such positions) form an orthogonal set of directions, and in many circumstances this can considerably simplify the data processing. Commercial three-sensor probes are available in this configuration, although these are optimally aligned for a probe stem parallel rather than normal to the flow. Single-sensor probes at this angle must be made specially.

The robustness of thermal sensors in turbomachine flows is a matter of considerable importance. Breakage seems to occur most frequently through dirt particle impingement, and in many industrial situations there is little that can be done to filter or otherwise clean up the air. Under these circumstances more success has been achieved with thin film probes, either as a spatial array on a solid insulating substrate, or as a fibre film. The principal disadvantages of these over a hot wire are a much reduced frequency response and the difficulty of making one's own probes.

DATA ACQUISITION

The principles of thermal anemometry are well known (see, for example, Bradshaw 1971 or Perry 1982), and are briefly summarised here. The sensor response is non-linear, and may be correlated by a variety of functions, the most common being King's Law. Figure 1(a) shows the response and equations for mean flow U and turbulent fluctuations as an rms quantity $\overline{u^2}$. It should be noted that the non-linearity distorts the transformation of the time-varying measured voltage $e(t)$ into the corresponding velocity $u(t)$, and for this reason this simple technique is not recommeded for turbulent intensities greater than a few percent.



$$U = \left(\frac{E^2 - A}{B}\right)^{1/n} \qquad U = kE_L$$

$$\overline{u^2} = \left(\frac{2E}{BnU^{n-1}}\right)^2 \overline{e^2} = \left(\frac{2UE}{(E^2 - A)n}\right)^2 \overline{e^2} \qquad \overline{u^2} = k\overline{e_L^2}$$

Fig. 1 Thermal anemometer response for (*a*) non-linear (*b*) linearised signals

The alternative shown in Fig. 1(*b*) is to linearise the measured voltage before it is converted into a velocity. This is done by applying a King's Law-type transformation to it. This can be done continuously before the signal is actually recorded and stored using appropriate hardware, or if the incoming signal is digitised and recorded discretely by means of a computer-based data logging system, the linearisation can be done point-by-point using software. Either way, a linearised voltage $e_L(t)$ results, and the turbulence quantities can be based on this. Linearisation is not a complete panacea, but depends on the accuracy to which the linearisation algorithm fits the actual response of the sensor. In this case hardware linearisers are more limited than software, there being less control over the form of the response equation and the number of coefficients which can be varied.

A major division of data acquisition techniques is of whether the time-varying signal (or the digitisation thereof) is to be saved, or the turbulent rms quantity is to be measured or computed on-line and saved without the full time history (Fig. 2). The limitation to saving the time-varying signal is the speed and size of data storage required. A sample rate of at least twice, and preferably several times, the cut-off frequency of the sensor is required to avoid aliasing (see Oldfield 1990). The sample record must be at least long enough to record the lowest frequency present which is of interest. For sensors of frequency response more than a few kHz, this implies fast digitisation and internal transfer to computer storage, and a large storage device (Moffat 1978).

The recent developments in computing have made this increasingly feasible, but there is still a case to be made for recording the mean and rms values of the measured signal using suitable instruments where the actual time-varying signal is of no interest. Such a system is very easy to set up, requires no special programming skills, and is relatively inexpensive to purchase and maintain. Its main drawback is that it seriously limits the data processing options available, as will be seen later.

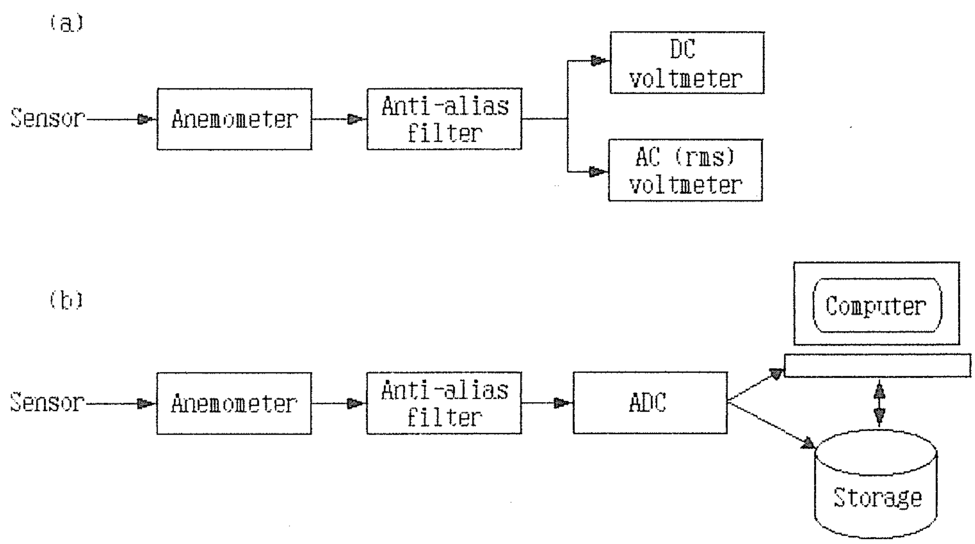


Fig. 2(*a*) Analogue (*b*) Digital data acquisition

DATA PROCESSING

On applying the sensor calibration and linearisation to the measured voltage, a velocity value is obtained. This value is a function of the angle between the sensor and the flow, and does not necessarily correspond to any of the velocity components of interest. It is known as the effective cooling velocity, U_e. Further data processing is required to reduce the several measurements of U_e made with different sensor directions at a single point in space to the velocity and turbulence components. There is a large literature describing this process, but the various methods appear to fall naturally into two categories, here termed the matrix and the geometric methods. These will be described here, with particular emphasis on the implications which these methods have for the probe design and method of data acquistion.

The matrix method

The basic geometry of a sensor in a turbomachine is shown in Fig. 3. The machine coordinate system is (x,y,z), with velocity components (U,V,W) parallel to these axes. (Conventionally the probe axis will be radial, so x is axial, y tangential and z radial, although the analysis is here kept quite general.) Another coordinate system is defined, which is attached to, and moves with, the sensor. This is (x',y',z'), with velocity components (V_x,V_y,V_z). Here x' is parallel to the sensor, y' is normal to the sensor and the probe axis, and z' is normal to the sensor and in the plane of the probe axis and the sensor.

The probe axis is parallel to the machine z-axis. A single slant sensor would be rotated about its own axis, or a three-sensor probe would simply have measurements made at several different sensor angles. A normal to the sensor

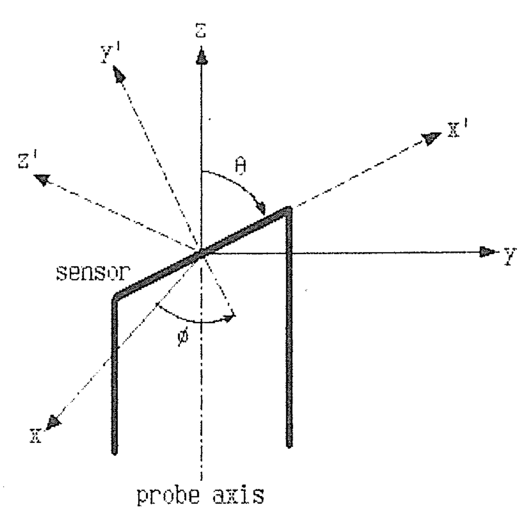


Fig 3 Machine and sensor axes for the matrix method of data processing

makes an angle ϕ with the x-axis. The sensor itself is inclined at angle θ to the probe axis. Thus the relation between the velocity components in the probe and laboratory coordinate systems is a simple matter of geometry

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} = \mathbf{N} \begin{bmatrix} U \\ V \\ W \end{bmatrix} \qquad (1)$$

$$\text{where } \mathbf{N} = \begin{bmatrix} -\sin\theta\sin\phi & \sin\theta\cos\phi & \cos\theta \\ -\cos\phi & -\sin\phi & 0 \\ \cos\theta\sin\phi & -\cos\theta\cos\phi & \sin\theta \end{bmatrix}$$

For any probe setting angle ϕ, (V_x, V_y, V_z) are unique. The velocity sensed by the probe is U_e, determined by a normal calibration of the sensor. This is related to the probe velocity components by

$$U_e^2 = k_T^2 V_x^2 + k_N^2 V_y^2 + V_z^2$$

where k_T is the sensitivity coefficient of the sensor to tangential velocities, and k_N is the sensitivity coefficient of the sensor to normal velocities. Hence

$$U_e^2 = \mathbf{K} \begin{bmatrix} V_x^2 \\ V_y^2 \\ V_z^2 \end{bmatrix} \qquad (2)$$

where $\mathbf{K} = [k_T^2 \; k_N^2 \; 1]$. Multiplying out the right-hand side of equation (1), squaring it and substituting in equation (2) gives

$$U_e^2 = \mathbf{KPT} \qquad (3)$$

where $\mathbf{P}$ =

$$\begin{bmatrix} \sin^2\theta\sin^2\phi & \sin^2\theta\cos^2\phi & \cos^2\theta & -2\sin^2\theta\sin\phi\cos\phi & 2\sin\theta\cos\theta\cos\phi & -2\sin\theta\cos\theta\sin\phi \\ \cos^2\phi & \sin^2\phi & 0 & 2\sin\phi\cos\phi & 0 & 0 \\ \cos^2\theta\sin^2\phi & \cos^2\theta\cos^2\phi & \sin^2\theta & -2\cos^2\theta\sin\phi\cos\phi & -2\sin\theta\cos\theta\cos\phi & 2\sin\theta\cos\theta\sin\phi \end{bmatrix}$$

and $\mathbf{T} = [U^2 \; V^2 \; W^2 \; UV \; VW \; UW]^{\mathrm{T}}$. In order to solve this equation for the components of $\mathbf{T}$, it is necessary to make six measurements of U_e at six different values of ϕ. Equation (3) can then be expanded into

$$[U_e^2] = \mathbf{QT}$$

where $[U_e^2]$ is the matrix of the measured velocities and $\mathbf{Q}$ is a 6×6 matrix whose elements are obtained by a $\mathbf{KP}$ multiplication for each value of ϕ. Thus the i'th row of $\mathbf{Q}$ comprises

(col.1)	$k_T^2 \sin^2\theta \sin^2\phi_i$	$+ k_N^2 \cos^2\phi_i$	$+ \cos^2\theta \sin^2\phi_i$
(col.2)	$k_T^2 \sin^2\theta \cos^2\phi_i$	$+ k_N^2 \sin^2\phi_i$	$+ \cos^2\theta \cos^2\phi_i$
(col.3)	$k_T^2 \cos^2\theta$		$+ \sin^2\theta$
(col.4)	$-2k_T^2 \sin^2\theta \sin\phi_i \cos\phi_i$	$+ 2k_N^2 \sin\phi_i \cos\phi_i$	$-2 \cos^2\theta \sin\phi_i \cos\phi_i$
(col.5)	$2k_T^2 \sin\theta \cos\theta \cos\phi_i$		$-2 \sin\theta \cos\theta \cos\phi_i$
(col.6)	$-2k_T^2 \sin\theta \cos\theta \sin\phi_i$		$+2 \sin\theta \cos\theta \sin\phi_i$

Theoretically, $\mathbf{T} = \mathbf{Q}^{-1}[U_e^2]$. However, if k_T, k_N and θ are constant, column 3 and the sum of colums 1 and 2 of $\mathbf{Q}$ are both constant, so that the matrix is degenerate and cannot be solved for any values of ϕ. This is contrary to what is reported in a number of papers which mention this technique without developing the analysis in detail.

The sensor calibration coefficients k_T and k_N are usually taken to be constant, with values of about 0.2 and 1.02 respectively (Jorgensen 1971). Even if an experimenter prefers to treat them as functions of velocity and ϕ, the variation from constant is unlikely to be sufficiently large to make the solution of $\mathbf{Q}$ a practical proposition. It is theoretically possible to solve $\mathbf{Q}$ if θ is not constant. For a multi-sensor probe this means an a non-axisymmetric arrangement of sensors, and a number of examples of this have been described (Acrivlellis 1980, Lakshminarayana 1981). This is clearly not possible with a rotated single-sensor probe.

Within these limits the technique is equally applicable to the time-mean velocities and the rms values of the fluctuating velocities, and does not require the measurement of the time-varying voltages.

If the sensor directions are orthogonal ($\theta = 54.7°$, $\Delta\phi = 120°$), the cross-product terms are eliminated in the above analysis and considerable simplifications are possible. In this case identities exist of the form

$$V_{x1} = V_{y2} = V_{z3}, \text{ etc}$$

where subscripts 1, 2 and 3 refer to the sensors (or sensor directions). A sensitivity matrix then arises (Jorgensen 1971)

$$\begin{bmatrix} U_{e1}^2 \\ U_{e2}^2 \\ U_{e3}^2 \end{bmatrix} = \mathbf{J} \begin{bmatrix} V_1^2 \\ V_2^2 \\ V_3^2 \end{bmatrix} \tag{4}$$

where $\mathbf{J} = \begin{bmatrix} k_T^2 & k_N^2 & 1 \\ 1 & k_T^2 & k_N^2 \\ k_N^2 & 1 & k_T^2 \end{bmatrix}$

and (V_1, V_2, V_3) are the velocities in the three orthogonal sensor directions. $\mathbf{J}$ is not singular for common values of the sensitivity coefficients, and so equation (4) can be solved for the orthogonal velocities and then equation (1) can be solved for U, V and W.

Using this method it is possible to process mean voltages in order to given mean velocities and the time-mean fluctuating voltages in order to give the turbulent components $\overline{u^2}$, $\overline{v^2}$ and $\overline{w^2}$. The cross-components of turbulent stress $\overline{uv}$, $\overline{vw}$ and $\overline{uw}$ can be obtained by combining the measured time-varying signals prior to processing, either by treating the recorded digitised signals or by combining the measured voltages $\overline{e}_1$, $\overline{e}_2$, $\overline{e}_3$ by analogue means to give $\overline{e_1 e_2}$, $\overline{e_2 e_3}$, $\overline{e_3 e_1}$ (Moffat 1978, Chew and Simpson 1988). The former technique requires fast sampling, as noted above, and a high accuracy of digitisation, particularly if the fluctuating component of voltage is small compared to the mean voltage. It relies essentially on making a point-by-point calculation of the instantaneous velocity vector, and then separating the mean and fluctuating velocity components from this. The turbulent quantities are derived from a statistical analysis of the latter.

The angle of the sensor appears to be fairly critical to the success of this method. Lekakis (1989) investigated the errors introduced when using the orthogonal sensor equations for non-orthogonal sensors and found that the errors introduced in the measured velocity increase rapidly as the sensor direction deviates from the orthogonal. A limit of 1 - 2° of allowable non-orthogonality was proposed, which demands a good quality of sensor manufacture and probe setting.

The geometric method

This method works on the basic trigonometric relations between the velocity vector and the sensor direction. In Fig. 4, the velocity vector is **U** and the sensor direction is **A**, lying in the x-z plane and making an angle θ (the slant angle) with the z-axis. The velocity vector makes a pitch angle β and a yaw angle γ with the x-z plane. The angle between **A** and **U** is α. The coordinate system is attached to the sensor, so that as the sensor direction changes by rotation about the z-axis, θ and β remain constant but γ and α vary. The relation between these angles is

$$\cos\alpha_i = \sin\theta\cos\beta\cos\gamma_i + \sin\theta\sin\beta \qquad (5)$$

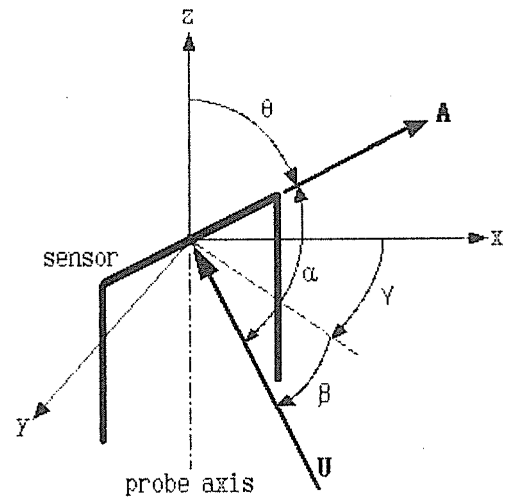


Fig. 4 Sensor and velocity vectors - the geometric method

The angular response of the sensor must now be expressed in the form

$$U_{ei}/U = f(\alpha_i, \beta, U) \qquad (6)$$

Champagne (1967) and Okiishi and Schmidt (1978) give calibration functions. Equations (5) and (6) constitute a set of six equations in six unknowns: $\alpha_{1,2,3}$, β, γ and U (the relation between the individual values of γ will be known from the sensor geometry and probe settings). These equations are non-linear and were originally solved by iteration (Hirsch and Kool 1977), but it has recently been demonstrated that they are capable of a quasi-analytical solution (Lekakis 1989). The method imposes some limits on the probe geometry and the choice of angles. The analysis is simplified if the sensor directions are orthogonal, but this is not essential to the method.

These equations are written in terms of the steady velocity. Their expansion into time-mean and time-averaged fluctuating components is a matter of great complexity, resulting in equations of 12 or more unknowns and many cross-products (Moffat 1978). Some simplification is possible if restrictions such as $v \ll u$ and $w \ll u$ are imposed, but this seriously limits the applicability of the solution. The method is only useful for turbulence measurements if a fast sampling technique is used and a point-by-point calculation of the instantaneous velocity vector is made, from which time-varying and turbulence quantities can be extracted.

CONCLUSIONS

The requirements of the complete measurement and analysis system must be taken into account when using a thermal anemometer to measure mean and turbulent quantities. Probably the most influential decision to be made is the choice of data acquisition technique. If an analogue method is chosen which gives time-mean and time-varying rms values only, this restricts the sensor geometry and the data processing technique. Under these circumstances it is not possible to use a non-orthogonal or a symmetric three-sensor probe, or a non-orthogonal single sensor probe. The use of a digital fast sampling technique enables the full time history of the signal to be recorded, and this increases the choice available. Such a system must be designed with careful regard for the bandwidth limits of the various components.

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