Session 3 - Hot Wire Measurements

A HOT WIRE MEASURING TECHNIQUE FOR MEAN VELOCITY AND REYNOLDS STRESS COMPONENTS IN COMPRESSIBLE FLOW

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SUMMARY

A technique for the measurement of mean velocity and Reynolds stress components with single sensor hot wire probes in compressible flow is described. The technique has been developed for the analysis of the three-dimensional turbulent flow downstream of turbine cascades in a wide velocity range, from incompressible to high subsonic flow.

This paper describes the method for the analysis of the hot wire signals, the calibration and measurement procedures and the problems encountered in the measurement of high velocity flows. The mathematical aspects of the numerical solution are examined and a detailed uncertainty analysis for the Reynolds stress measurement is given.

1. INTRODUCTION

Accurate knowledge of complex three-dimensional turbulent flows is often required for efficient design of many industrial devices, especialy in aeronautical and turbomachinery applications.

After the works of Fujita and Kovasznay [1] and Bissonnette and Mellor [2], a number of single sensor hot wire techniques have been developed for the measurement of mean velocity and Reynolds stress components in steady or periodic unsteady flows (see for instance [3-10]). All these techniques are based on the directional response of a heated wire exposed to a cooling flow with at least six different orientations. The methods differ each other for several important features such as the number of probes utilised, the way of probe insertion in the flow and the number of probe angular settings required for one complete measurement.

While the determination of mean velocity components is relatively straightforward and the measurement is in general accurate, the problem of measuring the Reynolds stress tensor is not simple, even in the case of incompressible flow. The mathematical aspects of a single sensor hot wire technique for incompressible flow, previously developed by the authors [10], are critically examined and some recommendations for its practical application are proposed. Then the problem of measurement of mean velocity and Reynolds stress tensor in compressible subsonic flows is discussed.

2. HOT WIRE ANALYSIS FOR INCOMPRESSIBLE FLOW

For a constant temperature hot wire anemometer without linearizer, the istantaneous effective cooling velocity ($\bar{q}+q'$) is related to the istantaneous anemometer output voltage ($\bar{e}+e'$) by the King's heat transfer law:

$$(\bar{e}+e')^2 = e_0^2 + B(\bar{q}+q')^n \tag{1}$$

where e_0, n, B are costants determined from the wire calibration.

The quantities $\bar{e}$ and $\sqrt{\overline{e'^2}}$ are the ones directly measured by means of a mean value and a true rms voltmeter. The following relationships for $\bar{q}$ and $\overline{q'^2}$, obtained from eq.(1) by application of the binomial theorem [11], can be adopted for relatively low turbulence intensity (less than 15%):

$$\bar{q} = \left(\frac{\bar{e}^2 - e_0^2}{B}\right)^{1/n}\left[1 + \frac{1}{n}\left(\frac{\overline{e'^2}}{\bar{e}^2 - e_0^2}\right) + \frac{2}{n}\left(\frac{1-n}{n}\right)\frac{\bar{e}^2\,\overline{e'^2}}{(\bar{e}^2 - e_0^2)^2}\right] \tag{2}$$

$$\overline{q'^2} = \left[2\bar{e}/\left(nB\bar{q}^{n-1}\right)\right]^2 \overline{e'^2} \tag{3}$$

The hot wire output depends not only on the flow speed, but on the wire orientation with respect to the flow as well. The Jørgesen's law [12] relates the effective cooling velocity to the velocity components u_t, u_n, u_b in the wire coordinate system (figure 1):

$$q^2 = u_n^2 + k^2 u_t^2 + h^2 u_b^2 \tag{4}$$

where h and k are sensitivity coefficients to be found by the probe directional calibration.

For a certain angle of rotation ϕ of the probe about its stem, the velocity components in the wire coordinate system (t,n,b) can be transformed into the fixed coordinate system (x_1,x_2,x_3) by

$$u_n = u_1 \cos\alpha + \left(u_2 \cos\phi - u_3 \sin\phi\right)\sin\alpha$$

$$u_t = -u_1 \sin\alpha + \left(u_2 \cos\phi - u_3 \sin\phi\right)\cos\alpha$$

$$u_b = u_2 \sin\phi + u_3 \cos\phi \tag{5}$$

To get a direct relationship between the cooling velocity and the velocity components u_1,u_2,u_3 in the fixed coordinate system, one substitutes eq. (5) in eq. (4) and obtains

$$q^2 = A_{11}u_1^2 + A_{22}u_2^2 + A_{33}u_3^2 + 2A_{12}u_1u_2 + 2A_{13}u_1u_3 + 2A_{23}u_2u_3 \tag{6}$$

where

$A_{11} = \cos^2\alpha + k^2\sin^2\alpha$

$A_{22} = \cos^2\phi\left(\sin^2\alpha + k^2\cos^2\alpha\right) + h^2\sin^2\phi$

$A_{33} = \sin^2\phi\left(\sin^2\alpha + k^2\cos^2\alpha\right) + h^2\cos^2\phi$

$A_{12} = \left[\left(1-k^2\right)\cos\phi\sin 2\alpha\right]/2$

$A_{13} = -\left[\left(1-k^2\right)\sin\phi\sin 2\alpha\right]/2$

$A_{23} = -\sin 2\phi\left(\sin^2\alpha + k^2\cos^2\alpha - h^2\right)/2$

are known coefficients, function of the wire angle α, of the rotation angle ϕ and of the Jørgesen's sensitivity coefficients h and k.

2.1 Solution for mean velocity components

Time averaging eq.(6) yields a relationship between mean cooling velocity $\bar{q}$ and mean velocity components $\bar{u}_1,\bar{u}_2,\bar{u}_3$, for each angular setting ϕ of the probe:

$$\bar{q}^2 = A_{11}\bar{u}_1^2 + A_{22}\bar{u}_2^2 + A_{33}\bar{u}_3^2 + 2A_{12}\bar{u}_1\bar{u}_2 + 2A_{13}\bar{u}_1\bar{u}_3 + 2A_{23}\bar{u}_2\bar{u}_3 + \text{turbulence terms} \tag{7}$$

In principle three readings with a slanted probe should be sufficient to determine the three unknown velocity components by solving a 3x3 non linear system of algebric equations. In practice, to obtain an acceptable accuracy (errors lower than ± 3% in the velocity and lower than ± 2 deg in yaw and pitch angles), one has to solve an overdetermined system of at least six non linear equations.

2.2 Solution for Reynolds stress components

When $\bar{u}_1$, $\bar{u}_3$, u_1', u_2', u_3' are at least one order of magnitude smaller than the primary velocity $\bar{u}_2$, eq.(6) can be rearranged in the form

$$\bar{q} + q' = \sqrt{A_{22}}\,\bar{u}_2 \quad [1+\Delta]^{1/2} \tag{8}$$

After expansion by the binomial theorem and linearization of eq.(8), an expression for the fluctuating cooling velocity can be obtained by linear decomposition. Squaring and time averaging the fluctuation q' yield an equation for the six Reynolds stress components:

$$\overline{q'^2} = \frac{A_{12}^2}{A_{22}}\overline{u_1'^2} + A_{22}\overline{u_2'^2} + \frac{A_{23}^2}{A_{22}}\overline{u_3'^2} + 2A_{12}\overline{u_1'u_2'} + 2\frac{A_{12}A_{23}}{A_{22}}\overline{u_1'u_3'} + 2A_{23}\overline{u_2'u_3'} \tag{9}$$

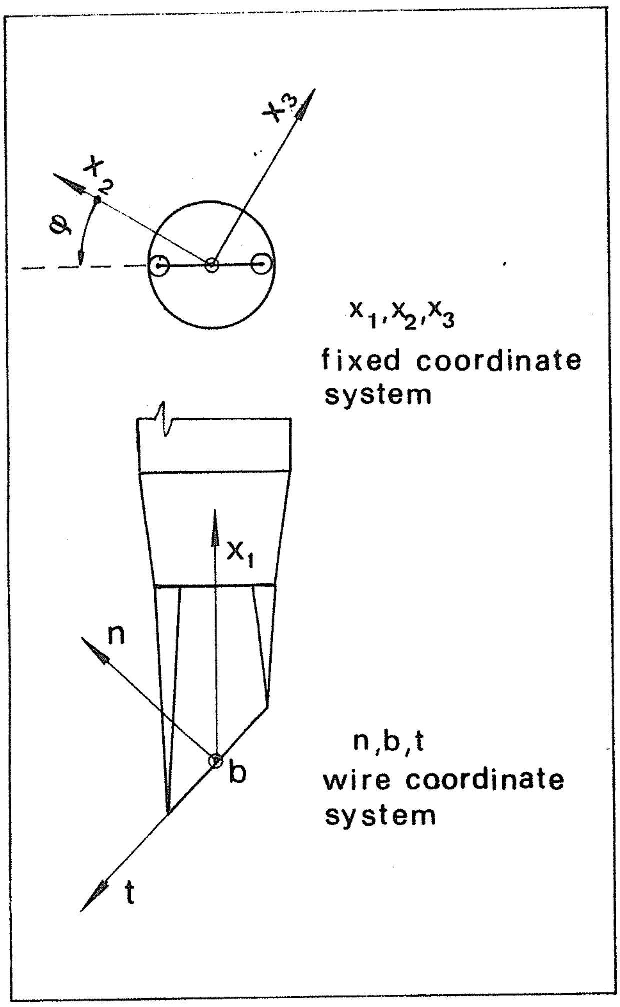

Figure 1 - Hot wire probe and reference coordinate systems.

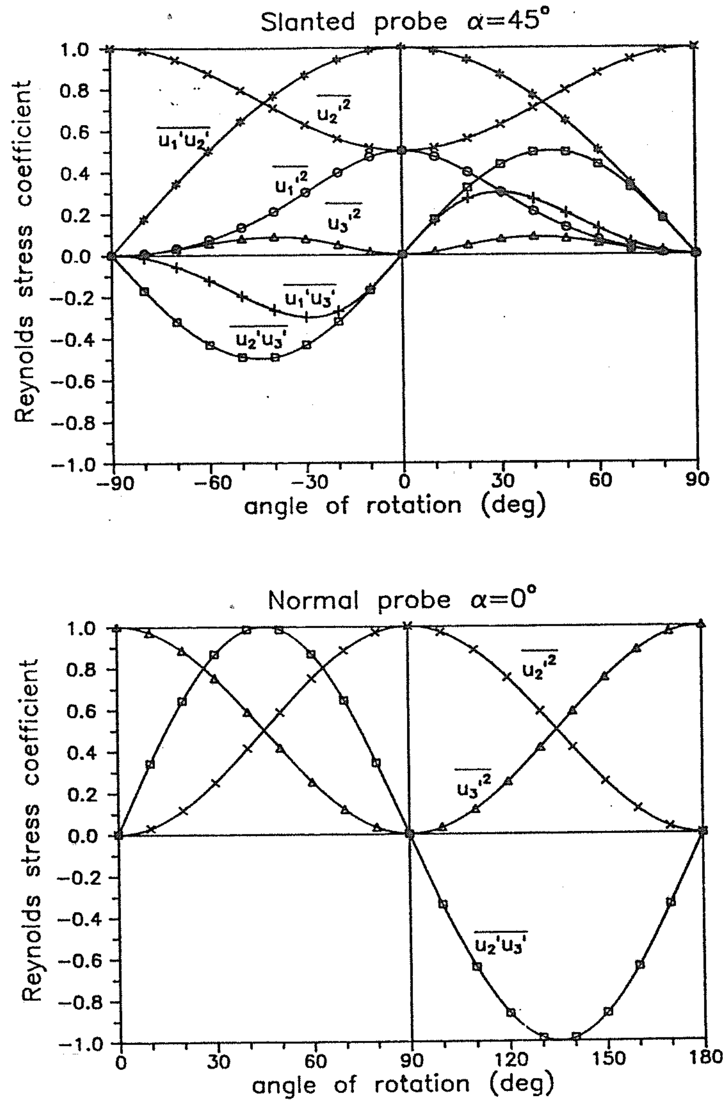

Figure 2 - Reynolds stress coefficient variations in function of the angle of rotation ϕ.

If yaw and pitch angles are greater than 10 deg, the drastic simplifications utilized for obtaining eq.(9) are not valid. In this case it is more convenient to rearrange eq.(6) in the form:

$$\bar{q} + q' = \sqrt{A_{11}\bar{u}_1^2 + A_{22}\bar{u}_2^2 + A_{33}\bar{u}_3^2 + 2A_{12}\bar{u}_1\bar{u}_2 + 2A_{13}\bar{u}_1\bar{u}_3 + 2A_{23}\bar{u}_2\bar{u}_3}\ \ [1+\Delta]^{1/2} \qquad (10)$$

Acting as before, but neglecting only the differences ($\overline{u_i'u_j'} - u_i'u_j'$), yields the following equation for the Reynolds stress components:

$$\overline{q'^2} = \left(B_1^2\overline{u_1'^2} + B_2^2\overline{u_2'^2} + B_3^2\overline{u_3'^2} + 2B_1B_2\overline{u_1'u_2'} + 2B_1B_3\overline{u_1'u_3'} + 2B_2B_3\overline{u_2'u_3'}\right)/B_m \qquad (11)$$

where

$$B_m = A_{11}\bar{u}_1^2 + A_{22}\bar{u}_2^2 + A_{33}\bar{u}_3^2 + 2A_{12}\bar{u}_1\bar{u}_2 + 2A_{13}\bar{u}_1\bar{u}_3 + 2A_{23}\bar{u}_2\bar{u}_3$$

$$B_1 = A_{11}\bar{u}_1 + A_{12}\bar{u}_2 + A_{13}\bar{u}_3 \qquad B_2 = A_{12}\bar{u}_1 + A_{22}\bar{u}_2 + A_{23}\bar{u}_3 \qquad B_3 = A_{13}\bar{u}_1 + A_{23}\bar{u}_2 + A_{33}\bar{u}_3$$

The B_i coefficients of eq.(11) are functions of the mean velocity components, while the coefficients of eq.(9) are not dependent on them.

Eq.(11) has been previously developed by Bridgeman, Sieverding and Borsboom [8].

For N angular settings of the probe, eq.(9) or eq.(11) gives a system of N algebric linear equations for the six unknown Reynolds stress components. In principle only six measurements, obtained from a slanted probe, are sufficient to solve the problem. In practice, even with a double number of data readings the results obtained are of scarce quality, expecially for the transverse and spanwise normal stress components.

Figure 2 shows the distribution of the coefficients of the Reynolds stress components versus the rotation angle ϕ for a slanted wire probe and a straight probe with $\overline{u_1}=\overline{u_3}=0$ and $h=1$, $k=0$ (in this case eq.(9) and eq.(11) are coincident). For the slanted probe the coefficient of the normal Reynolds component in transverse direction $\overline{u_3'^2}$ is about one order of magnitude lower than the other ones. Therefore this probe is rather insensitive to the $\overline{u_3'^2}$ component and a least square solution of the overdermined system of equations may be achieved with large errors in that component. The straight wire probe is not sensitive to $\overline{u_1'^2}$, $\overline{u_1'u_2'}$, $\overline{u_1'u_3'}$, but it is fairly sensitive to the $\overline{u_2'^2}$, $\overline{u_3'^2}$, $\overline{u_2'u_3'}$ stress tensor components.

A measuring technique was therefore developed, based on both straight and slanted wire probes outputs. Three measures are taken with a straight probe at ϕ = 45, 90, 135 deg and several measures with a slanted probe for more than five angular positions in the range $\phi = \pm 90$ deg.

The signal analysis is performed by solving separately the two systems of equations for the two probes. The $\overline{u_3'^2}$ component from the straight wire probe data is substituted in the overdetermined system of the slanted wire probe, that is solved with a least square procedure to obtain the remaining five unknown components. The comparison of the two stress components $\overline{u_3'^2}$ and $\overline{u_2'u_3'}$, obtained by both the probes, gives a sign of the measurement consistency.

2.3 Least square solution of the overdetermined systems of Reynolds stress equations

If one uses only a slanted probe, the overdetermined system (11) of N linear algebric equations for the six unknown Reynolds stress components can be rewritten as

$$\sum_{i=1}^{6} C_{ik} R_i = \overline{q_k'^2} \qquad k = 1, N \qquad (12)$$

with k index of the probe angular setting, i index of the Reynolds stress component R and C_{ik} influence coefficient. In matrix form the system is

$$C \cdot R = b$$

with C the N x 6 coefficient matrix, R the solution vector and b the vector of constants.

The least square technique reduces eq.(12) to a system of six equations in the six unknowns R_i

$$\sum_{k=1}^{N} \sum_{j=1}^{6} C_{jk} R_j C_{ik} = \sum_{k=1}^{N} C_{ik} \overline{q_k'^2} \qquad i = 1, 6 \qquad (13)$$

or

$$A \cdot R = C^T \cdot b$$

where $A = C^T \cdot C$ is the 6 x 6 matrix of the normal system.

For the proposed procedure based on the use of both slanted and straight wire probes, the overdetermined system of N equations in five unknowns for the slanted probe data is

$$\sum_{i=1}^{5} C_{ik} R_i = \overline{q_k'^2} - C_{6k} R_6 \qquad k = 1, N \qquad (14)$$

or

$$C_1 \cdot R = b_1$$

Applying the least square technique yields:

$$\sum_{k=1}^{N} \sum_{j=1}^{5} C_{jk} R_j C_{ik} = \sum_{k=1}^{N} C_{ik} \left(\overline{q_k'^2} - C_{6k} R_6 \right) \qquad i = 1, 5 \qquad (15)$$

or

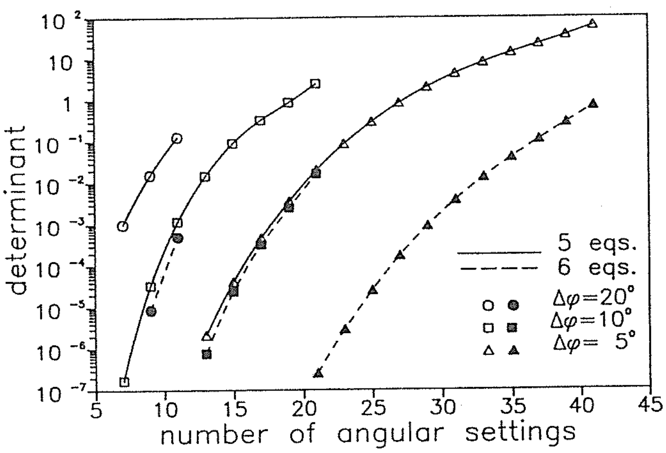

Figure 3 - Determinant of normal matrices.

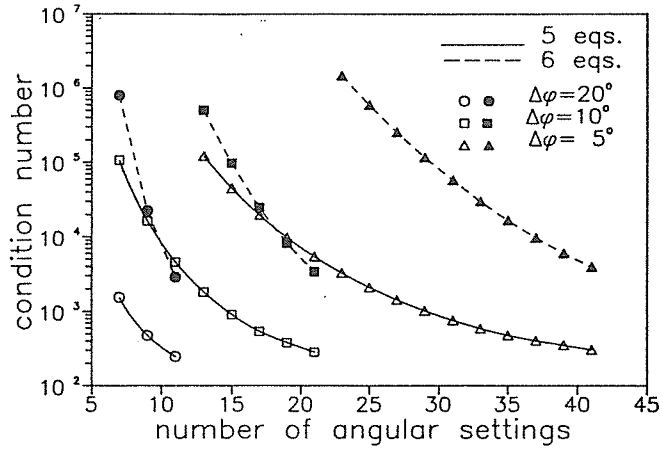

Figure 4 - Condition number of the normal matrices.

$$A_1 \cdot R = C_1^T \cdot b_1$$

with $A_1 = C_1^T \cdot C_1$ 5x5 matrix of the normal system.

The determinants of matrices A and A_1 are calculated in function of the number N of the probe angular settings (or data readings per measurement), for different values of $\Delta\phi$ and the results are shown in figure 3. Matrices A and A_1 are not singular, but in case of few angular settings they have small determinants, that increase as the number of readings increases. Det(A_1) is always larger than Det(A).

An index of the ill-conditioning of a system of equations is represented by the condition number defined as the product of the norm of the matrix of coefficients for the norm of its inverse

$$K(A) = ||A|| \cdot ||A^{-1}||$$

This number should be, as long as possible, near to the unity, to avoid ill-conditioning.

Furthermore a perturbation procedure shows that the relative change in the solution vector can be great as $K(A)$ times as much as the change in the vector of constants

$$\frac{||\delta b||}{||b||} / K(A) \leq \frac{||\delta R||}{||R||} \leq K(A) \cdot \frac{||\delta b||}{||b||}$$

The condition number has been calculated for both matrix A and A_1 in function of the number of probe angular settings adopted for one measurement and of the angular variation $\Delta\phi$ for each probe rotation.

Figure 4 shows that the procedure based on the use of two probes gives rise to a better-conditioned system and that the well-conditioning of the two systems depends on the width of the angular range from which the data are taken, rather than on the number of probe readings.

2.4 Sensitivity analysis of the normal systems of equations

The foregoing analysis, based on the condition number, simply bounds the relative error of the solution vector caused by a change in the vector of constants. To quantify the effect of errors in the reading of the hot wire rms outputs on each Reynolds stress component, a sensitivity analysis has been performed.

In case of measurements taken only with a slanted wire probe, by differentiating system (13) with respect to each root mean squared fluctuating output, one obtains N systems of 6 linear equations with a total of 6xN unknowns $\partial R_i/\partial e_k'$. Assuming a random distribution of ± 1 mV error over each fluctuating output e_k' and taking the average over 1000 events, one gets an estimation of the mean error in each Reynolds stress component in percent of τ_w/ρ.

$$\frac{\partial R_i}{\tau_w/\rho} = \left(\frac{1}{\tau_w/\rho}\right)\frac{1}{10^3}\sum_{j=1}^{10^3}\sqrt{\sum_{k=1}^{N}\left[\frac{\partial R_i}{\partial e_k}\cdot \mathrm{rnd}(\pm 1\,\mathrm{mV})\right]^2} \qquad e_k{}' = \sqrt{\overline{e_k{}'^2}} \qquad i = 1,6 \qquad (16)$$

A similar procedure can be applied for the measuring technique based on the use of two probes. In that case one has to differentiate system (15) with respect to each e_k' and R_6 and obtains N+1 systems of 5 linear equations with 5x(N+1) unknowns $\partial R_i/\partial e_k$' and $\partial R_i/\partial R_6$. As before, one can estimate the error on each Reynolds stress component in per cent of τ_w/ρ:

$$\frac{\partial R_i}{\tau_w/\rho} = \left(\frac{1}{\tau_w/\rho}\right)\frac{1}{10^3}\sum_{j=1}^{10^3}\sqrt{\sum_{k=1}^{N}\left(\frac{\partial R_i}{\partial e_k}\cdot \mathrm{rnd}(\pm 1\,\mathrm{mV})\right)^2 + \left(\frac{\partial R_i}{\partial R_6}\Delta R_6\right)^2} \quad i = 1,5 \qquad (17)$$

$$\Delta R_6 = \frac{1}{10^3}\sum_{j=1}^{10^3}\sqrt{\sum_{k=1}^{3}\left(\frac{\partial R_6}{\partial e_k}\cdot \mathrm{rnd}(\pm 1\,\mathrm{mV})\right)^2}$$

The results of figure 5 show that the procedure based on the use of two probes not only reduces considerably the error on the normal stress component in transversal direction $\overline{u_3'^2}$, as expected, but also in $\overline{u_1'^2}$ and $\overline{u_1'u_2'}$ components. A number of three readings for the straight probe and of nine readings for the slanted probe in the range ϕ = ± 80 deg appears adequate to keep errors within reasonable limits.

2.5 Probe directional calibration

Due to large condition numbers, the solution of system (15) is rather sensitive to small changes in the matrix of coefficients. Therefore a carefull calibration of the directional factors h and k is needed to reduce systematic errors.

Generally, if the measurement is performed aligning the probe stem with the stream direction, the pitch factor h is assumed to depend on the pitch angle ϑ and the yaw factor k on the yaw probe angle β [12] (figure 6a). In this case the probes are not sensitive to the roll angle ϕ and the calibration can be obtained simply by rotating the stem of a ϑ angle in the plane of the prongs (thus inducing a β variation) and in the plane perpendicular (resulting in a ϑ variation), as illustrated in figure 6b.

When the stem is perpendicular to the mean flow, the probe presents roll sensitivity because the prongs assume different orientations with respect to the flow during rolling from -90 to +90 deg.

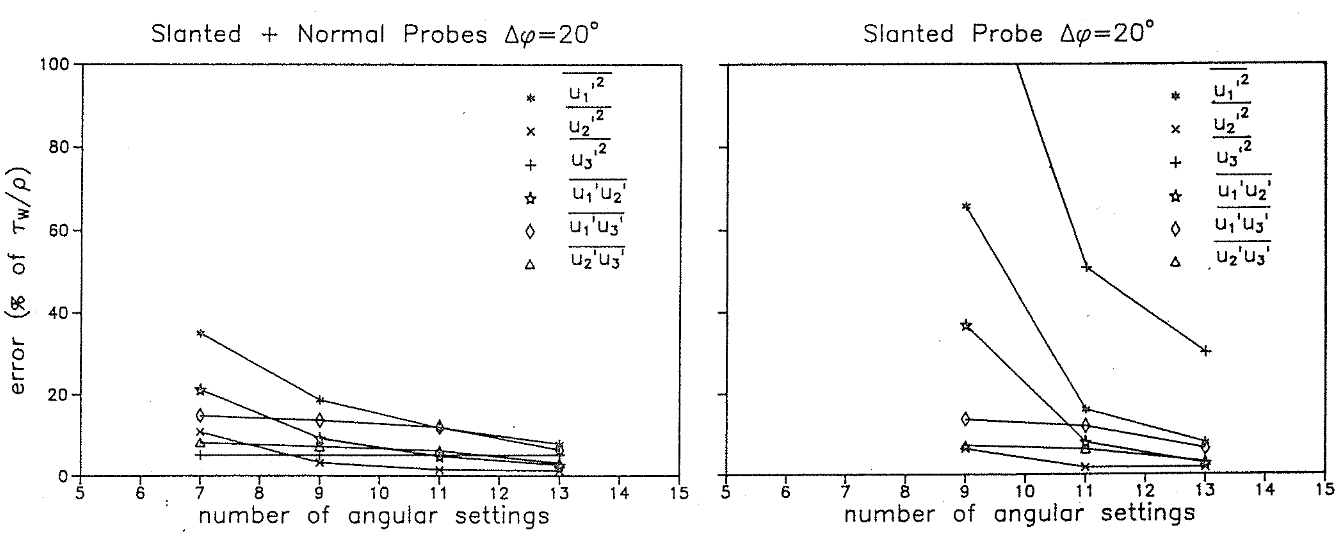

Figure 5 - Error in Reynolds components for a random distribution of ± 1 mV error in rms output voltage (τ_w/ρ = 1 m/s)

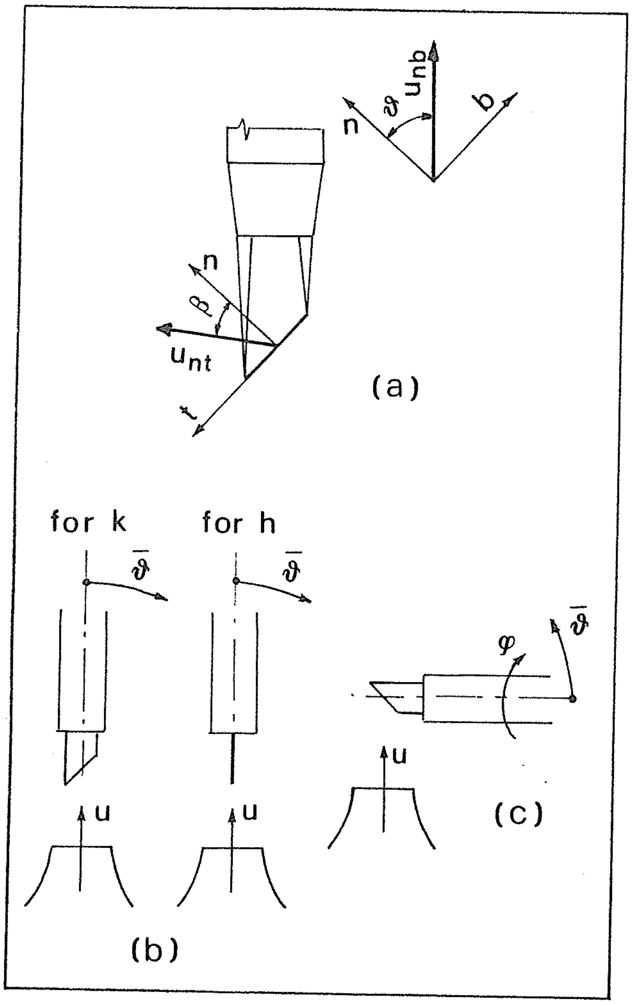

Figure 6 - (a) Definition of β and ϑ angles; (b) directional calibration for measurements with the probe aligned with the mean flow; (c) directional calibration for measurements with the probe perpendicular to the mean flow.

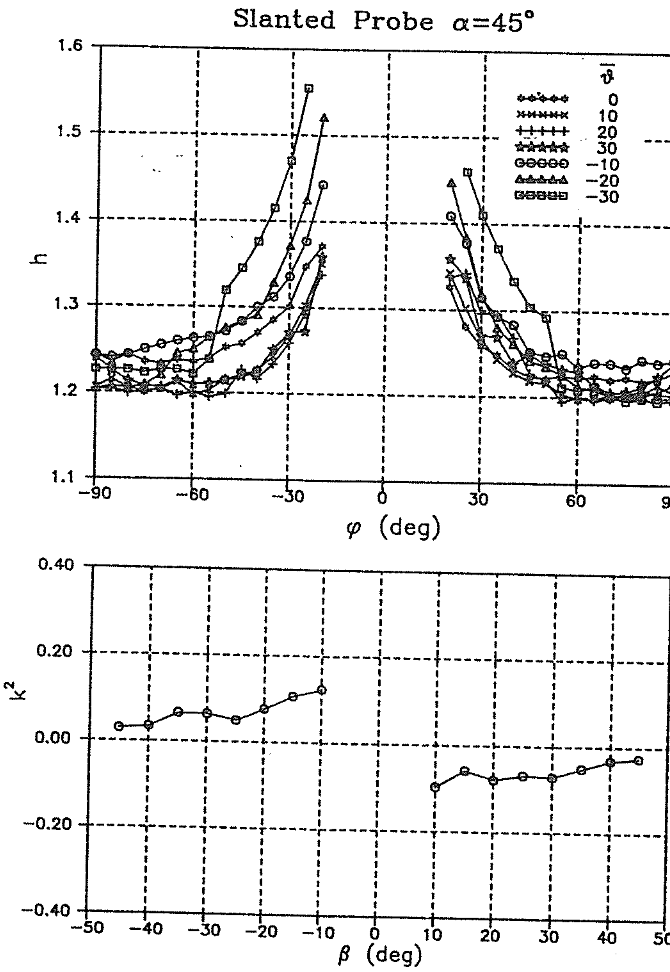

Figure 7 - Directional calibration results for h and k^2 coefficients.

Therefore the directional calibration has to be performed reproducing conditions which are representative of the ones prevailing during the flow measurements, with variations of both roll angle ϕ and pitch angle $\bar{\vartheta}$ (figure 6c).

Typical results of this directional calibration are shown in figure 7. Negative values for k^2, apparently unrealistic, are usual for slanted probes, because of the cooling effect of the inclined prongs when the flow is normal to the wire.

2.6 Measurements in a two-dimensional incompressible turbulent boundary layer

The two-dimensional incompressible turbulent boundary layer developing on the lateral wall of a low speed wind tunnel has been used to get a direct evaluation of the hot wire technique for Reynolds stress measurement.

In figure 8 the results show a reasonable agreement with the reference data of Klebanoff [13]. As expected, the normal as well as shear stresses associated with the velocity fluctuations perpendicular to the surface present a higher degree of scattering according to the foregoing equation sensitivity analysis and to the estimates of uncertainty of Löfdahl [14].

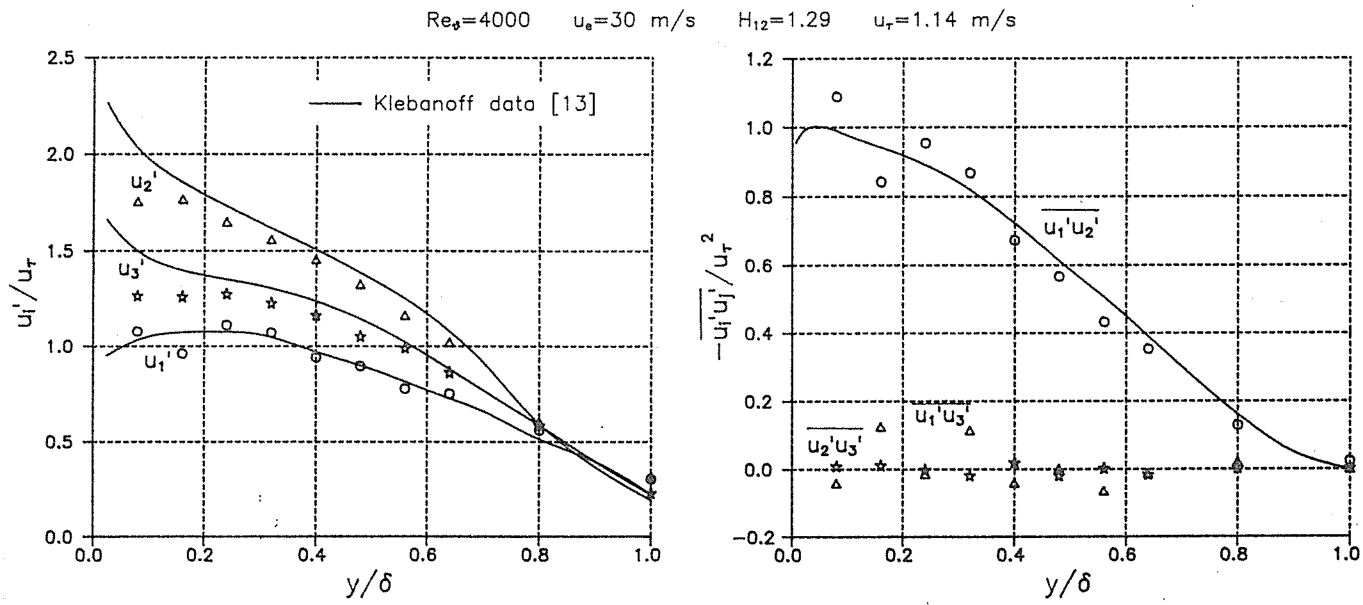

Figure 8 - Turbulence measurements in an incompressible turbulent boundary layer: comparison with Klebanoff results (u_1', u_2', u_3' fluctuating velocity components in spanwise, streamwise and transversal directions).

3. HOT WIRE ANALYSIS FOR COMPRESSIBLE SUBSONIC FLOW

The heat transferred from a heated wire to the flow is expressed, in terms of dimensionless groups, by the following law [15]:

$$Nu = f(Re, \tau) \tag{18}$$

where $Re = \rho u d/\mu$ is the wire Reynolds number and $\tau = (T_w - T_e)/T_e$ is the temperature loading.

The ratio of the temperature T_e and the total temperature T_t is a function of the Mach number [16]; it can be obtained by performing resistance measurements with the unheated wire placed in a flow where total temperature and Mach number are known.

For a constant temperature anemometer, assuming adiabatic flow, the temperature loading τ is nearly costant and eq.(18) reduces to a linear relationship between the Nusselt number and the square root of the Reynolds number [17]. This relationship can be rewritten in terms of hot wire output voltage and mass flux to obtain an equation equivalent to the King's law (1) for the case of compressible adiabatic flow:

$$e^2/(T_w - T_e) = A' + B'(\rho u)^n \tag{19}$$

3.1 Solution for mean velocity components

By applying the same procedure employed in the case of incompressible flow, one gets a relationship for the mean effective cooling mass flux in function of the anemometer outputs for each probe angular setting:

$$\overline{\rho q} = \left(\frac{\bar{e}^2 - A''}{B''}\right)^{1/n}\left[1 + \frac{1}{n}\left(\frac{\overline{e'^2}}{\bar{e}^2 - A''}\right) + \frac{2}{n}\left(\frac{1-n}{n}\right)\frac{\bar{e}^2\overline{e'^2}}{(\bar{e}^2 - A'')^2}\right] \tag{20}$$

with $A'' = A'(T_w - T_e)$, $B'' = B'(T_w - T_e)$

Using a modified Jørgensen law with mass flux components in place of intrinsic velocity components and carrying out the same analysis as in the incompressible case yield a system of N non linear equations in the three unknown $\overline{\rho u_1}$, $\overline{\rho u_2}$, $\overline{\rho u_3}$

$$\overline{\rho q}^2 = A_{11}\overline{\rho u_1}^2 + A_{22}\overline{\rho u_2}^2 + A_{33}\overline{\rho u_3}^2 + 2A_{12}\overline{\rho u_1}\,\overline{\rho u_2} + 2A_{13}\overline{\rho u_1}\,\overline{\rho u_3} + 2A_{23}\overline{\rho u_2}\,\overline{\rho u_3} \qquad (21)$$

If mean total temperature and static pressure are known from separate measurements and the mean mass flux is approximated with the product of density and velocity mean values, the velocity components $\overline{u}_i$, static temperature $\overline{T}$ and density $\overline{\rho}$ can be calculated from the known values of mean mass flux components by using the relationship between total and static temperature and the perfect gas law:

$$\frac{\overline{T}_t}{\overline{T}} = 1 + \frac{\gamma - 1}{2}\left(\frac{\overline{\rho u_1}^2 + \overline{\rho u_2}^2 + \overline{\rho u_3}^2}{\gamma \overline{p}^2}\right)R\overline{T} \qquad (22)$$

$$\overline{\rho} = \overline{p}/R\overline{T} \qquad (23)$$

3.2 Solution for Reynolds stress components

According to Kovasznay [15], the fluctuating voltage signal results from a linear combination of density, velocity and total temperature fluctuations:

$$\frac{e'}{\overline{e}} = S_\rho \frac{\rho'}{\overline{\rho}} + S_u \frac{u'}{\overline{u}} + S_T \frac{T_t'}{\overline{T}_t} \qquad (24)$$

If $M \geq 1.2$ (Morkovin [16]) or also if $M < 1.2$ in case of $\tau > 0.5$ and Re > 20 (Horstman and Rose [18]), the density sensitivity coefficient S_ρ is equal to the velocity sensitivity coefficient S_u and eq.(24) becomes:

$$\frac{e'}{\overline{e}} = S_{\rho u} \frac{(\rho u)'}{\overline{\rho u}} + S_T \frac{T_t'}{\overline{T}_t} \qquad (25)$$

For temperature loadings τ larger than one, the total temperature coefficient S_T becomes smaller than the mass flux coefficient [18]. Under these conditions and for nearly adiabatic flow (where mean total temperature gradients and, therefore, also total temperature fluctuations are small) the wire senses only the mass flux fluctuations. As for the incompressible case a relationship holds between the mean squared fluctuation of effective cooling mass flux and the mean squared fluctuation of output voltage

$$\overline{(\rho q)'^2} = \left[2\overline{e}/\left(nB''\overline{(\rho q)}^{n-1}\right)\right]^2 \overline{e'^2} \qquad (26)$$

and one can derive a linear equation that relates the mean squared fluctuations of effective cooling mass flux to the six temporal correlations of the fluctuating mass flux components in the fixed reference system, for each probe angular setting:

$$\overline{(\rho q)'^2} = C_{11}\overline{(\rho u_1)'^2} + C_{22}\overline{(\rho u_2)'^2} + C_{33}\overline{(\rho u_3)'^2} + C_{12}\overline{(\rho u_1)'(\rho u_2)'} +$$
$$+ C_{13}\overline{(\rho u_1)'(\rho u_3)'} + C_{23}\overline{(\rho u_2)'(\rho u_3)'} \qquad (27)$$

System (27) can be solved with the same numerical procedure as in the incompressible case to get the six unknowns mass flux correlations $\overline{(\rho u_i)'(\rho u_j)'}$.

At this stage one has to separate the mean squared density fluctuations $\overline{\rho'^2}$ and the Reynolds stress components $\overline{u_i'u_j'}$ from the mass flux correlations. The first step is approximating the six mass flux correlations in terms of mean squared density fluctuation, fluctuating velocity and density-velocity correlations:

$$\overline{(\rho u_i)'(\rho u_j)'} = \overline{\rho'^2}\,\overline{u}_i\overline{u}_j + \overline{\rho}\,\overline{u}_i\overline{\rho' u_j'} + \overline{\rho}\,\overline{u}_j\overline{\rho' u_i'} + \overline{\rho}^2\overline{u_i'u_j'} \qquad (28)$$

The density-velocity correlations can be expressed in terms of the fluctuating velocity correlations by using the following relationship between the istantaneous values of total and static temperature

$$\overline{T}_t + T_t' = \overline{T} + T' + \frac{1}{2C_p} \sum_{i=1}^{3} \left(\overline{u}_i^2 + u_i'^2 + 2\overline{u}_i u_i' \right) \tag{29}$$

Multiplying eq.(29) by u_j', time averaging the resulting equation and assuming negligeable total temperature fluctuations (adiabatic flow), one obtains a relationship between the fluctuating temperature-velocity correlations and the Reynolds stress components:

$$\overline{T'u_j'} = -\frac{1}{C_p} \sum_{i=1}^{3} \overline{u}_i \overline{u_i'u_j'} \tag{30}$$

Pressure, density and static temperature fluctuating components are related by the perfect gas law:

$$\frac{p'}{\overline{p}} = \frac{\rho'}{\overline{\rho}} + \frac{T'}{\overline{T}} \tag{31}$$

Assuming that the relative pressure fluctuations are small, compared to the relative static temperature and density fluctuations [18,19], and combining eq.(30) and eq.(31), one finally gets:

$$\overline{\rho'u_j'} = \frac{\overline{\rho}}{\overline{T}C_p} \sum_{i=1}^{3} \overline{u}_i \overline{u_i'u_j'} \tag{32}$$

Introducing eq.(32) in eq.(28) leads to a system of six algebric linear equations in the seven unknown $\overline{u_i'u_j'}$ and $\overline{\rho'^2}$.

$$\overline{(\rho u_i)'(\rho u_j)'} = \overline{\rho'^2}\,\overline{u}_i\overline{u}_j + \overline{\rho}^2\overline{u_i'u_j'} + \frac{\overline{\rho}^2}{\overline{T}C_p}\left[\overline{u}_i \sum_{k=1}^{3} \overline{u}_k \overline{u_k'u_j'} + \overline{u}_j \sum_{k=1}^{3} \overline{u}_k \overline{u_i'u_k'} \right] \tag{33}$$

One more equation for $\overline{\rho'^2}$ in terms of $\overline{u_i'u_j'}$ is needed. By separating in eq.(29) fluctuating from mean terms, with the assumption $T_t' = 0$, one gets

$$T' = -\frac{1}{C_p} \sum_{i=1}^{3} \overline{u}_i u_i' \tag{34}$$

Squaring and time averaging eq.(34) and combining it with eq.(31) yield

$$\overline{\rho'^2} = \frac{\overline{\rho}^2}{C_p^2 \overline{T}^2} \sum_{i=1}^{3} \sum_{j=1}^{3} \overline{u}_i \overline{u}_j \overline{u_i'u_j'} \tag{35}$$

that is the seventh equation needed to solve the problem of determining the six Reynolds stress components and the mean squared fluctuating density.

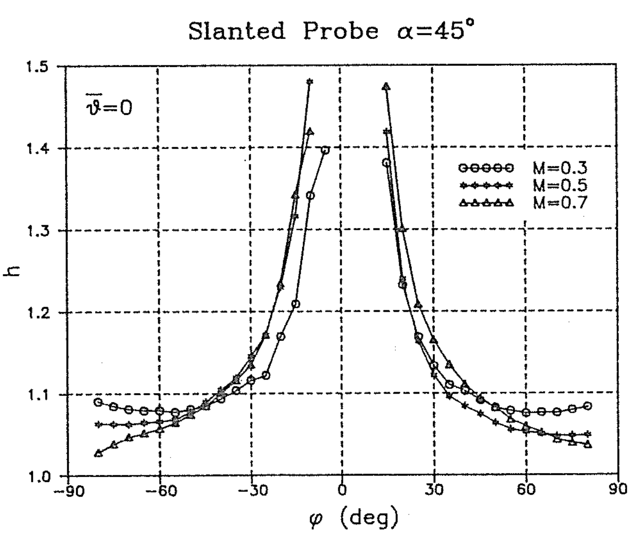

Figure 9 - Directional calibration of h coefficient for different Mach numbers.

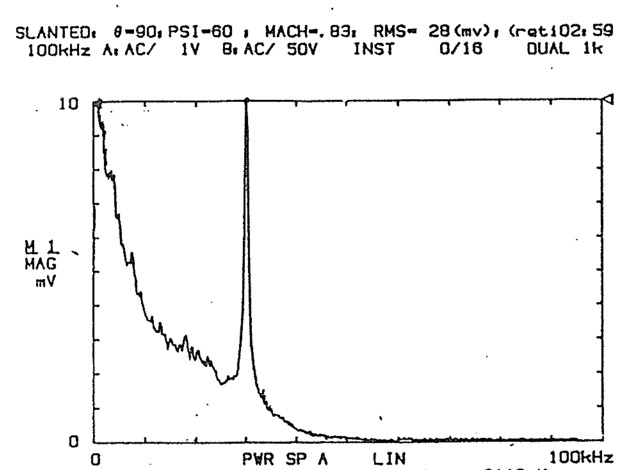

Figure 10 - Spectrum of anemometer output voltage.

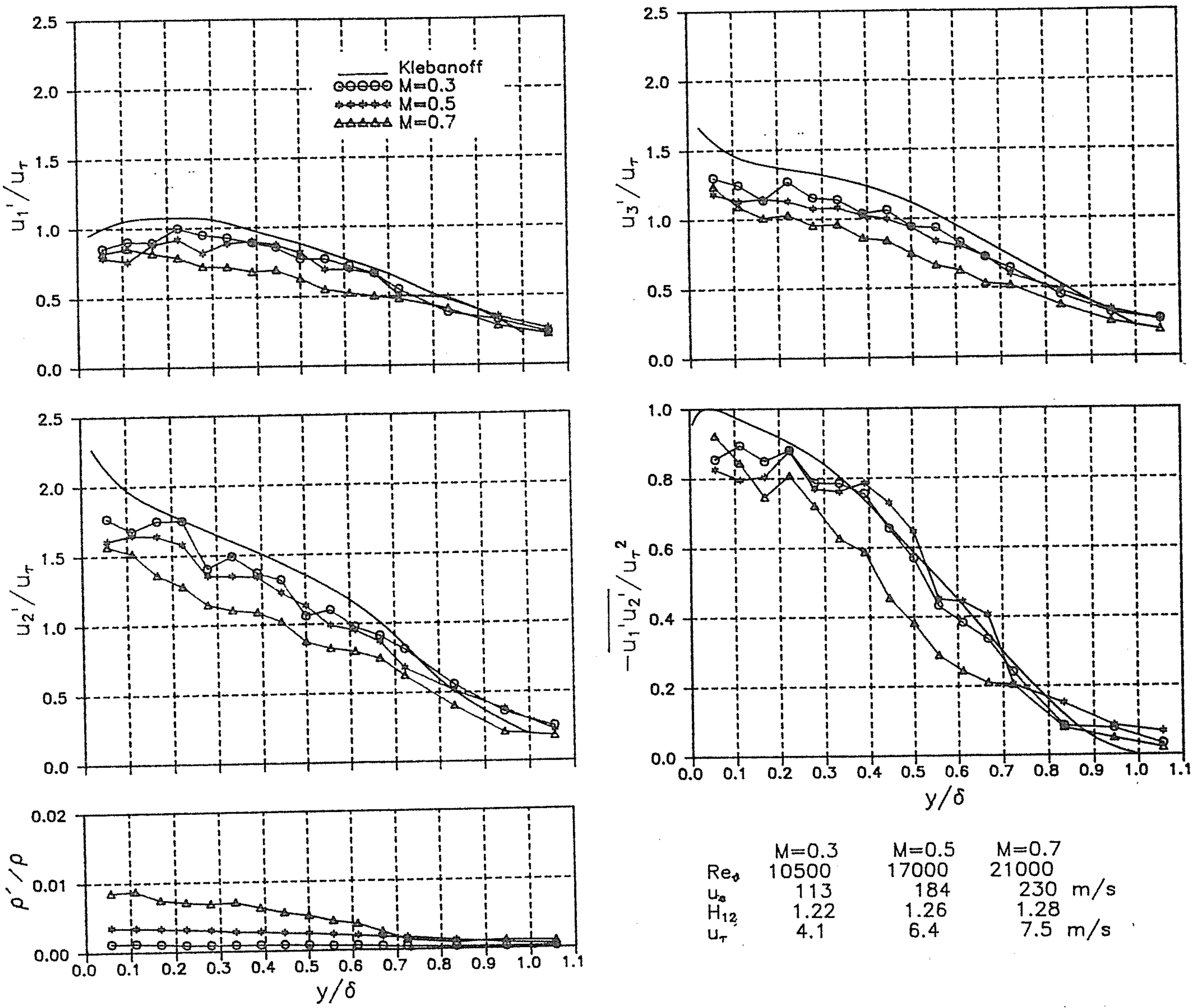

Figure 11 - Turbulence measurements in a compressible turbulent boundary layer: comparison with Klebanoff results (u_1', u_2', u_3' fluctuating velocity components in spanwise, streamwise and transversal directions).

3.3 Application of the hot wire technique for compressible flow

The application of the described hot wire technique to the measurement of high subsonic flows brings pratical problems not encountered in low speed measurements. Very efficient air filtering systems are required to avoid wire breakage; adequate pass band of the hot wire anemometer is needed to resolve the highest frequencies of turbulent fluctuations, which increase with the flow speed; automated data acquisition and probe positioning systems are needed, especially for intermittent tunnels, to reduce air consumption and overall time for the experiment.

Furthermore the wire must be calibrated in the range of Reynolds numbers of interest and the calibration of the directional factors h and k have to be performed for different Mach numbers. For example figure 9 shows the Mach number effect on the directional calibration of h coefficient for a slanted wire probe.

Finally a direct inspection of the hot wire voltage output by means of an on line spectrum analyzer is recommended to identify possible strain gauging phenomena, due to vortex shedding from the support needles. Commercial single wire probes were found to be affected by severe strain gauge effects when introduced with the stem normal to the main flow at high subsonic Mach numbers, as shown in figure 10, where a sharp peak of energy is associated with a frequency of about 30 kHz. The probes were therefore modified by reducing the needle lenght; by this way the energy peaks in the range 0-100 kHz have disappeared. A low pass filter at 100 kHz was applied to cut high frequency noise.

Reynolds stress measurements have been performed in two-dimensional turbulent boundary layers developing on the lateral wall of a transonic wind tunnel for different Mach number ranging from 0.3 to 0.8. The results shown in figure 11 are more scattered compared with the incompressible results of figure 8 and present a trend for a slow decrease of the fluctuation levels with Mach number. Nevertheless the overall distribution is in agreement with the measurements of Klebanoff [13] in incompressible flow. This decreasing trend, shown also by the results of Kistler [19] for a large Mach number range in supersonic flows, in our case is probably due to the fact that the boundary layer is not completely in equilibrium, because of the streamwise pressure gradient in the upstream contraction. This effect is increasing with the Mach number.

4. CONCLUSIONS

A hot wire technique has been developed for measuring mean velocity and Reynolds stress tensor in turbulent incompressible flows. The analysis of signals from single sensor probes rotated about their stems for different angular settings leads to overdetermined systems of algebric equations to be solved with least square techniques.

A detailed sensitivity analysis shows that the Reynolds stress system solution can be rather inaccurate, if the data are taken from a not wide enough range of angular probe positions and that the combined use of slanted and straight wire probes results in better-conditioned coefficient matrices and, therefore, in a lower sensitivity of the solution to small changes in voltage readings. Authors' experience suggests the need of an accurate directional calibration of the wire directional sensitivity factors, expecially when the probe is introduced normal to the flow.

Classic assumptions, on which are based methods for determining temperature, density and streamwise velocity fluctuations in turbulent supersonic and transonic flows, have been incorporated in the described hot wire technique to extend it to the measurement of Reynolds stress components in turbulent compressible flows. An algebric equation system has been developed, to separate the density fluctuations from the Reynolds stress components in the six fluctuating mass flux correlations resulting from the least square solution of the hot wire equations.

The technique has been verified in two-dimensional compressible boundary layers up to M=0.8 and has been used to study the three-dimensional turbulent flow developing downstream of turbines cascades for different exspansion ratios.

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NOMENCLATURE

b, n, t	wire coordinate system (figure 1)
C_p	specific heat at costant pressure
d	hot wire diameter
e	istantaneous hot wire voltage
h, k	directional sensitivity coefficients of the Jørgensen's law
H_{12}	boundary layer shape factor
M	Mach number
n	calibration exponent of King's law
N	number of probe angular settings
Nu	hot wire Nusselt number

p, p_t	static pressure, total pressure
q	istantaneous effective wire cooling velocity
R	gas costant
Re_ϑ	momentum thickness Reynolds number
T, T_t	static temperature, total temperature
T_e	wire equilibrium temperature
T_w	hot wire temperature
u_b,u_n,u_t	velocity components in the wire coordinate system b,n,t (figure 1)
u_e	freestream velocity
u_1,u_2,u_3	velocity components in the fixed coordinate system x_1,x_2,x_3 (figure 1)
u_τ	wall friction velocity $u_\tau = (\tau_w/\rho)^{1/2}$
x_1,x_2,x_3	fixed coordinate system (figure 1)
α	wire angle
β	angle between velocity and n direction in nt plane (figure 6a)
γ	ratio of specific heats
δ	boundary layer thickness
ϑ	inclination of the probe stem during directional calibration (figures 6b and 6c)
μ	viscosity
ρ	density
τ	temperature loading
τ_w	wall shear stress
ϕ	probe rotation angle about the probe axis

Superscripts

—	time averaged
'	fluctuating component

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