ACCURACY OF INTEGRATED MASS FLOWS OBTAINED FROM TURBOMACHINE TRAVERSES

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N.B. Wood

Central Electricity Research Laboratories,
TPRD, CEGB,
Leatherhead, KT22 7SE, UK.

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ABSTRACT

Very high accuracy is sometimes claimed for mass flows obtained by integration of radial traverse data obtained in turbomachines. An error assessment method is described and the results are presented for a sample turbine data set in order to stimulate discussion on the accuracies achieved by other groups. For the examples presented here the probable errors in integrated mass flow vary between approximately 2.5% and 20% (99% probability).

NOTATION (Conventions as shown in Fig. 3)

C	velocity
f_m	mass flow function $r \rho C \sin \alpha \cos \bar{\Phi}$
g	$(1 + \frac{\gamma-1}{2} M^2)/(\gamma M^2)$
g'	$g \left[1 - (1 + \frac{\gamma-1}{2} M^2)^{-\frac{\gamma}{\gamma-1}} \right]$
$\dot{m}$	mass flow rate
M	Mach number
p	static pressure
p_T	total pressure
r	radial position of probe in turbine annulus
r_c	turbine annulus casing radius
r_h	turbine annulus hub radius
s	non-dimensional standard deviation or standard erorr
S	dimensional standard deviation or standard error
α	flow yaw angle (in axial-tangential plane)
γ	ratio of specific heats of flow medium
ρ	density of flow medium
$\bar{\Phi}$	flow pitch angle (in radial plane containing the flow direction)

1. INTRODUCTION

An important parameter in the determination of turbine performance is the annulus mass flow rate, which cannot accurately be measured independently when testing operating steam turbines. CERL have made radial flow traverses in a number of operating large steam turbines using the pressure probe system described by Langford, Keeley and Wood (1982) (see Fig. 1), and the mass flow rate can be derived from the measurements so obtained. In this note, the parameters affecting the accuracy of the mass flow so derived are examined and an estimate of the accuracy is made for a typical data set.

The mass flow is obtained from the radial traverse stations downstream from moving blade rows; the flow here is unsteady but may be considered axially symmetrical, wakes from upstream nozzle rows having substantially 'filled'. The final stage exit flow may in some cases exhibit a more general circumferential non-uniformity imposed by the exhaust ducting. A typical installation of traverse access ways in a turbine is shown in Fig. 2.

Mean flow Mach numbers at the stations in question are from 0.2 to 0.3 except at the exhaust where 0.5 to 0.8 is more typical.

In assessing accuracy one must consider the effects of both systematic and random errors. Apart from the possible circumferential non-uniformities already referred to, the other main systematic error may arise from the effects of flow unsteadiness on the response of the probes. This has two components; one involves the response of the pressure lines between the pressure orifices in the flow and the transducer, whilst the other results from the flow pattern in the external flow. These have been the subjects of separate investigations (e.g. Wood, Langford and Priddy, 1982) and only random errors will be considered here.

2. DEPENDENCE OF MASS FLOW ON MEASURED QUANTITIES

The mass flow $\dot{m}$ is obtained from the following integral:

$$\dot{m} = 2\pi \int_{r_h}^{r_c} \rho\, C \sin\alpha \cos\bar{\phi}\, r\, dr = 2\pi \int_{r_h}^{r_c} f_m\, dr \quad \ldots (1)$$

where the angle conventions are shown in Fig. 3, ρ is density, C is velocity, α is swirl angle (measured from tangential direction, positive in the direction of rotation of the rotor blades), $\bar{\phi}$ is the flow pitch angle, r is radius; subscripts h and c are at hub and casing respectively.

Logarithmic differentiation of the integrand, f_m gives

$$\frac{df_m}{f_m} = \frac{d\rho}{\rho} + \frac{dC}{C} + \frac{d(\sin\alpha)}{\sin\alpha} + \frac{d(\cos\bar{\phi})}{\cos\bar{\phi}} + \frac{dr}{r} \quad \ldots (2)$$

The velocity, C, is obtained from the ratio of total to static pressure p_T/p, i.e.

$$C = \left\{ \frac{2\gamma}{\gamma-1} \frac{p}{\rho} \left[\left(\frac{p_T}{p}\right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \right\}^{\frac{1}{2}} \qquad \ldots (3a)$$

where γ is the ratio of specific heats. With the probe system shown in Fig. 1 p_T and p are measured independently. For this case, logarithmic differentiation of eq. (3a) gives

$$\frac{dC}{C} = \frac{1 + \frac{\gamma-1}{2} M^2}{\gamma M^2} \frac{dp_T}{p_T} - \frac{1 - \frac{1}{2} M^2}{\gamma M^2} \frac{dp}{p} - \frac{1}{2} \frac{d\rho}{\rho} \qquad \ldots (3b)$$

(Differential measurement of p_T-p is considered in Section 4.) Of the components of eq. (3b), the pressure terms are known, but the density ρ requires further investigation.

For a perfect gas, or as an approximation for superheated steam, we may write

$$\rho = \frac{p}{RT}$$

so that

$$\frac{d\rho}{\rho} = \frac{dp}{p} - \frac{dT}{T} \qquad \ldots (4a)$$

In the case of wet steam, with which we are principally concerned here,

$$\rho \approx \rho_g/(1-Y)$$

where ρ is the mixture density, ρ_g is the density of the gas phase and Y is the wetness mass fraction.

Therefore,

$$\frac{d\rho}{\rho} = \frac{d\rho_g}{\rho_g} + \frac{Y}{1-Y} \frac{dY}{Y}$$

From the Clausius - Clapeyron equation

$$\rho_g \simeq \frac{T_{sat}}{h_{fg}} \frac{dp}{dT_{sat}}$$

where h_{fg} is the latent heat of condensation and T_{sat} is the saturation temperature. Over a small range

$$\frac{dp}{dT_{sat}} = \text{const.} \frac{p}{T_{sat}}$$

$$\therefore \rho_g \simeq \frac{T_{sat}}{h_{fg}} \left(\text{const.} \frac{p}{T_{sat}}\right) = \text{const.} \frac{p}{h_{fg}}$$

$$\therefore \frac{d\rho_g}{\rho_g} = \frac{dp}{p} - \frac{dh_{fg}}{h_{fg}}$$

It can be shown that, for low pressure steam,

$$\frac{dh_{fg}}{h_{fg}} \simeq 0.0165 \frac{dp}{p}$$

$$\therefore \frac{d\rho_g}{\rho_g} \simeq 0.98 \frac{dp}{p} \simeq \frac{dp}{p}$$

and $$\frac{d\rho}{\rho} \simeq \frac{dp}{p} + \frac{Y}{1-Y} \frac{dY}{Y} \qquad \text{... (4b)}$$

For simplicity we will write equations (4a) and (4b) as a single equation

$$\frac{d\rho}{\rho} = \frac{dp}{p} - 2t \qquad \text{... (4)}$$

where $t = \frac{1}{2}(dT/T)$ for superheated gas and

$t = -\frac{1}{2} \frac{Y}{1-Y} \frac{dY}{Y}$ for wet steam.

Substituting eq. (4) into eq. (3b) gives

$$\frac{dC}{C} \approx \frac{1 + \frac{\gamma-1}{2} M^2}{\gamma M^2} \left(\frac{dp_T}{p_T} - \frac{dp}{p}\right) + t \qquad \ldots (3)$$

Substituting (3) and (4) in (2) gives

$$\frac{df_m}{f_m} \approx \frac{1 + \frac{\gamma-1}{2} M^2}{\gamma M^2} \frac{dp_T}{p_T} - \frac{1 - \frac{\gamma-1}{2} M^2}{\gamma M^2} \frac{dp}{p} + \frac{d(\sin\alpha)}{\sin\alpha} + \frac{d(\cos\bar{\phi})}{\cos\bar{\phi}} + \frac{dr}{r} - t$$

$$= g(M) \frac{dp_T}{p_T} - [g(M) - 1] \frac{dp}{p} + \alpha\cot\alpha \frac{d\alpha}{\alpha} - \bar{\phi}\tan\bar{\phi} \frac{d\bar{\phi}}{\bar{\phi}} + \frac{dr}{r} - t \qquad \ldots (5)$$

In the case of low pressure wet steam, typical relaxation lengths are large compared with probe dimensions, so M and γ are defined for "frozen" or gas phase conditions.

3. ESTIMATION OF ACCURACY OF MASS FLOW DETERMINATION

The probable error is given by the square root of the sum of the squares of the individual terms of equation (5) (Kline and McLintock, 1953).

It is now necessary to estimate the magnitudes of the individual terms. This will be done for a typical set of data obtained in a low pressure turbine cylinder.

3.1 Pressures

The errors affecting measurement of both total and static pressure arise from the degree of precision of the transducer calibration and from variation in turbine flow conditions. During the experiments repeated reference measurements were made of pressure e.g. in accessway C or D in Fig. 2. The variation in this measurement will incorporate errors from both sources. Therefore, knowing the standard error s_{tr} of the regression line through the transducer calibration and the standard deviation s_p about the mean of the reference pressure variation, the standard deviation s_f of the turbine flow condition is given by

$$s_p^2 = s_f^2 + s_{tr}^2$$

Each of these refers by definition to a 68.3% probability of values lying within the confidence interval expressed by s. In practice it is considered that a 99% probability is more useful and this requires specification of 2.6s as the confidence interval (Benedict and Wyler, 1979).

In the following discussion, s will denote the standard deviation non-dimensionaliseed by the mean pressure level $\bar{p}$ and S will denote the standard deviation expressed in pressure units.

The pressures at accessways C and D, used as reference, were both at a similar level, 22-25 kPa, and it will be assumed that the non-dimensional flow variation s_f was similar for the other traverse locations at different pressure levels. By contrast, the standard error for the calibration is expressed as a fixed confidence interval in pressure units, leading to different percentage errors for different pressure levels. Therefore s_f is found from eq. (6) for accessways C/D and the combined error s_p is then calculated by recombining s_f and s_{tr} for traverse locations at other pressure levels.

We have the transducer regression line standard error

$$S_{tr} = 0.50 \text{ mb}$$

and the reference pressure standard deviation

$$S_p = 1.0 \text{ mb with pressure level 22 kPa}$$

i.e.

$$s_{tr} = 2.2 \times 10^{-3}$$

$$s_p = 4.5 \times 10^{-3}$$

and from eq. (6)

$$s_f = 3.9 \times 10^{-3}$$

These are for 68.3% probability; in the remaining discussion 99% probability values will be used.

Table 1 shows the standard deviations for the different traverse positions. (Position E is downstream from a fixed blade row, so mass flow could not be obtained from a single radial traverse. The data are included for interest.)

Traverse	Pressure Level (kPa)	68.3% Probability			99% Probability $s_{p99} = 2.6\ s_p$
		s_{tr}	s_f	s_p	
A	100	5×10^{-4}	3.9×10^{-3}	3.9×10^{-3}	1.0×10^{-2}
B	50	1×10^{-3}	3.9×10^{-3}	4.0×10^{-3}	1.0×10^{-2}
C	22	2.3×10^{-3}	3.9×10^{-3}	4.5×10^{-3}	1.2×10^{-2}
D	25	2×10^{-3}	3.9×10^{-3}	4.4×10^{-3}	1.2×10^{-2}
$E(p_T)$	22	2.3×10^{-3}	3.9×10^{-3}	4.5×10^{-3}	1.2×10^{-2}
E(p)	12	4.2×10^{-3}	3.9×10^{-3}	5.7×10^{-3}	1.5×10^{-2}
$F(p_T)$	5	1×10^{-2}	3.9×10^{-3}	1.1×10^{-2}	2.9×10^{-2}
F(p)	3	1.7×10^{-2}	3.9×10^{-3}	1.7×10^{-2}	4.4×10^{-2}

Table 1: Estimation of Pressure Errors

For the static pressure there is an additional uncertainty arising from the degree of precision with which the aerodynamic calibration of the static pressure disc can be obtained. It has not been posible to make a statistical analysis of this, but the 99% probability confidence interval s_{c99} is estimated to be 2×10^{-3} for the subsonic part of the calibration ($M < 0.8$). The transonic part of the calibration (Langford, Keeley and Wood, 1982) is subject to greater uncertainty and the confidence interval for this regime ($0.8 < M < 1.3$) is estimated to be 2.5×10^{-2} for 99% probability. These figures may have to be revised in the light of the results of the European Workshop on Probe Calibrations (Broichhausen and Fransson, 1984).

3.2 Flow Angles

The resolution of turbine yaw angle measurement via the CERL disc probe is usually given as ±0.5° and can reasonably be taken as a confidence interval for 99% probability. The effect of this error on $s_{\alpha 99} = d(\sin\alpha)/\sin\alpha$ varies with the angle measured as shown in Fig. 5. If the flow is axial $s_{\alpha 99}$ is negligible at 0.004% but for $\alpha \sim 60°$, which is representative of some of the traverses, the error increases to 1/2%. In the case of nozzle exit traverses, where $\alpha \sim 15°$, $s_{\alpha 99}$ is 3% for 0.5°. However, nozzle exit stations are not used for estimation of mass flow because of blockage effects and circumferential variation.

The resolution for the pitch angle measurement is given as ±1°. The pitch angle Φ tends to be near zero and seldom rises beyond ±15°. This would give a maximum error in $d(\cos\Phi)/\cos\Phi$ of 0.5% (5×10^{-3}).

3.3 Radial Position

The error in setting and reading radial position from the calibrations on the probe support shaft may be up to 0.2 cm with a mean radius, r of order 1 m. Therefore, we will assume

$$\left(\frac{dr}{r}\right)_{99} \approx 2 \times 10^{-3} .$$

3.4 Errors in Measurement of Temperature or Wetness

The accuracy of temperature measurement is probably better than 0.04°C to 99% probability (Volluz, 1961).

Therefore

$$\frac{dT}{T} = 0\ (10^{-4})$$

It is difficult to estimate the accuracy of the measurement of wetness, as obtained from the optical probe (Walters, 1985), but it is thought to be as good as the measurement of temperature.

We can say, therefore, that

$$\frac{Y}{1-Y}\frac{dY}{Y} = 0\ (10^{-5}) \quad .$$

3.5 Total Error

Because the probable errors are estimated by squaring and adding the component terms of eq. (5), the smaller terms make a negligible contribution to the overall result. Thus, the terms $d(\cos\bar{\phi})/\cos\bar{\phi}$, dr/r and t may be neglected, leaving solely the pressure and swirl angle terms to be considered. From these, the total errors for our example traverses have been estimated as follows.

Traverse	Mean Mach. No.	g(M)	Mean α (deg)	$s_{\alpha 99}$	$dp_T/p_T = s_{p99}$	$dp/p = (s_{p99}^2+s_{c99}^2)^{1/2}$	$\frac{df_m}{f_m}$
A	0.22	16	62	4.5×10^{-3}	1.0×10^{-2}	1.0×10^{-2}	2.2×10^{-1}
B	0.25	12	70	3×10^{-3}	1.0×10^{-2}	1.0×10^{-2}	1.6×10^{-1}
C	0.35	6.2	60	5×10^{-3}	1.2×10^{-2}	1.2×10^{-2}	9.7×10^{-2}
D	0.32	7.4	40	1×10^{-2}	1.2×10^{-2}	1.2×10^{-2}	1.2×10^{-1}
E	1.10	0.75	17	2.7×10^{-2}	1.2×10^{-2}	2.9×10^{-2}	2.9×10^{-2}
F	1.00	0.88	90	4×10^{-5}	2.9×10^{-2}	5.1×10^{-2}	2.6×10^{-2}

Table 2: Estimation of Mass Flow Error for Independent Measurement of p_T and p

The term S_{c99} is the estimated probe calibration error.

Therefore the predicted mass flow accuracy (from the final column of Table 2) varies from 22% to 2.6% depending on flow conditions in the traverse, with a root mean square average value (for the moving blade exit traverses) of 14%. For comparison, the mass flow values obtained for the moving blade exit traverses were as follows:

Traverse	Mass Flow $\dot{m}$ (kg/s)	$\frac{\dot{m}-\dot{m}_F}{\dot{m}_F}$
A	72.72	1.3×10^{-1}
B	65.07	1.4×10^{-1}
C	71.05	1.1×10^{-1}
D	72.54	1.3×10^{-1}
F	64.19	-
Mean	69.11	-

Table 3: Mass Flows Obtained from Integrated Traverse Data

Third column shows variation from Traverse F, which from Table 2 was expected to have the lowest error.

The standard deviation about the mean is 6.0%, with a range of +5.2%, -7.1%. The error for 99% probability (2.6 s.d.'s) is 15.6%, which compares well with the average predicted value of 14%.

It should be noted that for the example cited, the traverse downstream from the final stage gave the best accuracy (±2.6% for 99% probability). However this is not a general result, depending as it does on the particular levels of Mach number and flow angles, and each data set should be assessed separately using the system adopted here. Further, turbine exit traverses can be subject to non-axisymmetry imposed by the exhaust.

4. USE OF DIFFERENTIAL PRESSURE MEASUREMENT

Because the errors in the derivation of velocity from separate total and static pressure measurements are potentially so large in low subsonic flow, it is worth examining the errors arising if the velocity is derived from a differential pressure measurement, $p_T - p$.

Equation (3a) may be written as

$$c^2 = \frac{2\gamma}{\gamma-1}\frac{p}{\rho}\left[\left(\frac{p_T-p}{p}+1\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \qquad \ldots (3c)$$

giving

$$\frac{dC}{C} = \frac{1}{2}\left(\frac{dp}{p} - \frac{d\rho}{\rho}\right) + g(M)\left(1 - \frac{p}{p_T}\right)\left(\frac{d\Delta p}{\Delta p} - \frac{dp}{p}\right)$$

$$= g'(M)\left(\frac{d\Delta p}{\Delta p} - \frac{dp}{p}\right) + t \qquad \ldots (6)$$

where $\Delta p = p_T - p$

and $g'(M) = g(M)\left(1 - \frac{p}{p_T}\right)$

$$= g(M)\left[1 - \left(1 + \frac{\gamma-1}{2} M^2\right)^{-\frac{\gamma}{\gamma-1}}\right]$$

Hence eq. (5) becomes

$$\frac{df_m}{f_m} = g'(M)\frac{d\Delta p}{\Delta p} + \left[1 - g'(M)\right]\frac{dp}{p} + \frac{d(\sin\alpha)}{\sin\alpha} + - - \ldots (7)$$

The variation of g'(M) with M is shown in Fig. 4, from which it can be seen that it is very much less than g(M) at low Mach numbers, and remains lower even at supersonic Mach numbers. Thus the multiplying factor on the pressure error is far smaller for low subsonic Mach numbers than that on the pressure errors occurring for independent measurements. The stability of differential pressure transducers has in the past been found inferior to that of equivalent absolute transducers. It remains to investigate more fully the relative accuracies and the characteristics of a multi-hole probe system which is suitable for differential measurements.

For the traverses quoted we may investigate the likely mass flow errors for comparison with Table 2. For this comparison it has been assumed that the calibration error for the differential transducer, s_{tr} is twice that for the absolute transducer. The results are shown in Table 4.

Traverse	Mean Mach No.	g'(M)	Mean α(deg)	$s_{\alpha 99}$	$\frac{d\Delta p}{\Delta p}$	dp/p $=(s_{p99}^2+s_{c99}^2)^{1/2}$	$\frac{df_m}{f_m}$
A	0.22	0.50	62	4.5×10^{-3}	1.0×10^{-2}	1.0×10^{-2}	8.4×10^{-3}
B	0.25	0.49	70	3×10^{-3}	1.1×10^{-2}	1.0×10^{-2}	8.0×10^{-3}
C	0.35	0.48	60	5×10^{-3}	1.6×10^{-2}	1.2×10^{-2}	1.1×10^{-2}
D	0.32	0.48	40	1×10^{-2}	1.4×10^{-2}	1.2×10^{-2}	1.4×10^{-2}
E	1.10	0.39	17	2.7×10^{-2}	1.6×10^{-2}	2.9×10^{-2}	3.3×10^{-2}
F	1.00	0.41	90	4×10^{-5}	5.3×10^{-2}	5.1×10^{-2}	3.7×10^{-2}

Table 4: Estimation of Mass Flow Error for Differential Pressure Measurement

Probe calibration errors as Table 2. Transducer calibration error (differential) assumed twice the error of absolute transducer

Comparison with Table 2 shows the potential superiority of the differential pressure measurement at low Mach numbers. At the higher Mach numbers the assumption of greater calibration errors for the differential transducer has made the estimated total error greater than for the independent pressure measurements. It remains to gain further experience with differential transducers and so test the above assumption.

5. CONCLUSIONS

The accuracy of derivation of mass flow from the present CERL turbine traverse probe system has been examined and estimated for a particular data set. The precision in relation to random errors has been presented for 99% probability (i.e. 2.6 standard errors) and values between 22% and 2.6% have been obtained depending principally on mean pressure level, Mach number and flow yaw angle.

For the sample data set used here the analysis showed that the most accurate estimate of mass flow was obtained from the traverse downstream of the final stage. Assuming flows here are axisymmetric, mass flow estimates were within ±3% for 99% probability.

The analysis also showed that considerable improvement in mass flow measurement accuracy at the upstream stations may be possible using a different probe system suitable for differential pressure measurement.

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7. ACKNOWLEDGEMENTS

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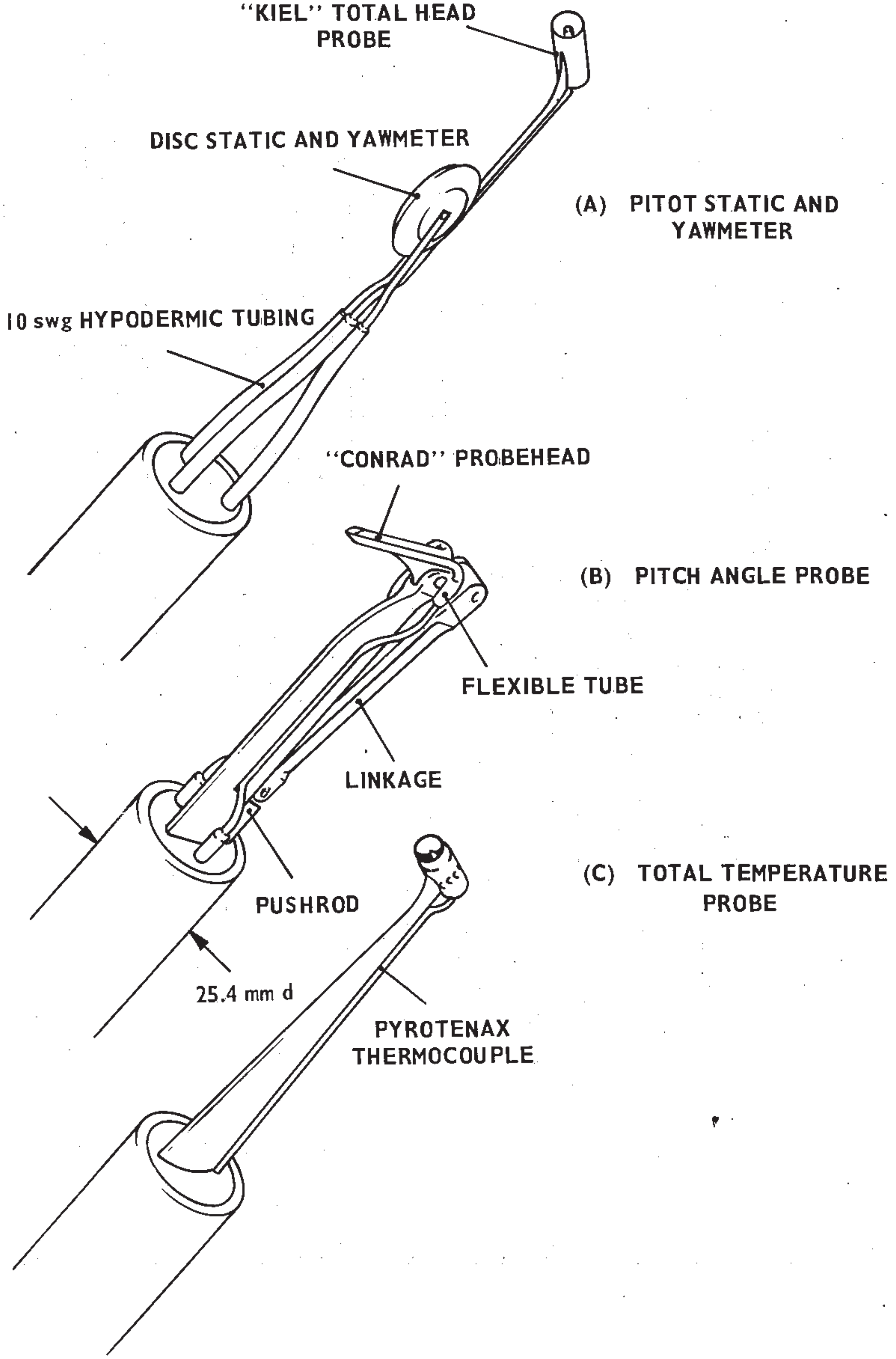
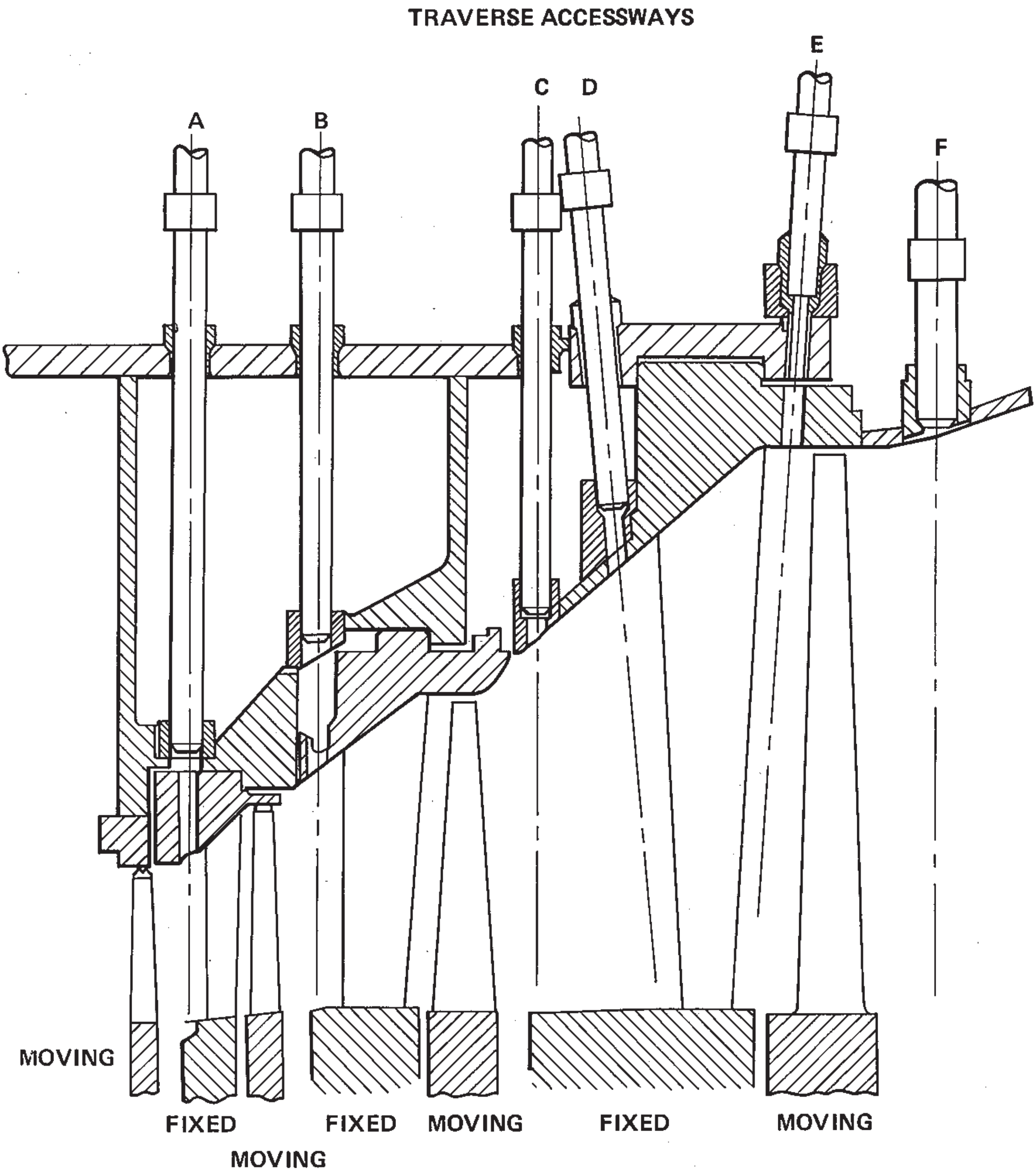


FIG. 1 CERL TURBINE TRAVERSING PROBES



CROSS-SECTION OF INNER CYLINDER IN PLANES OF ACCESSWAYS

FIG. 2 LAYOUT OF TRAVERSE ACCESSWAYS IN LOW PRESSURE TURBINE CYLINDER

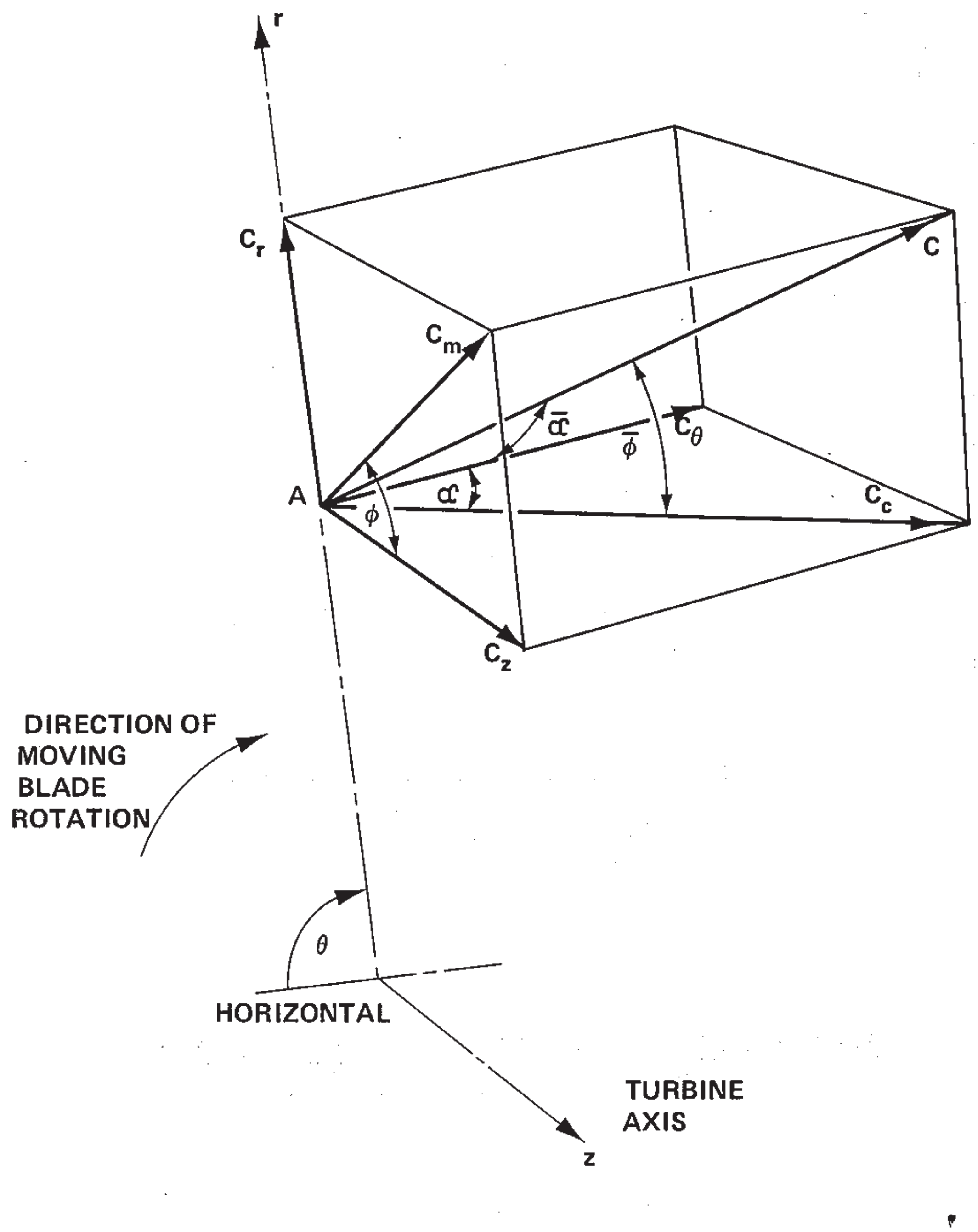


FIG. 3 CONVENTION FOR FLOW ANGLES AND VELOCITY COMPONENTS

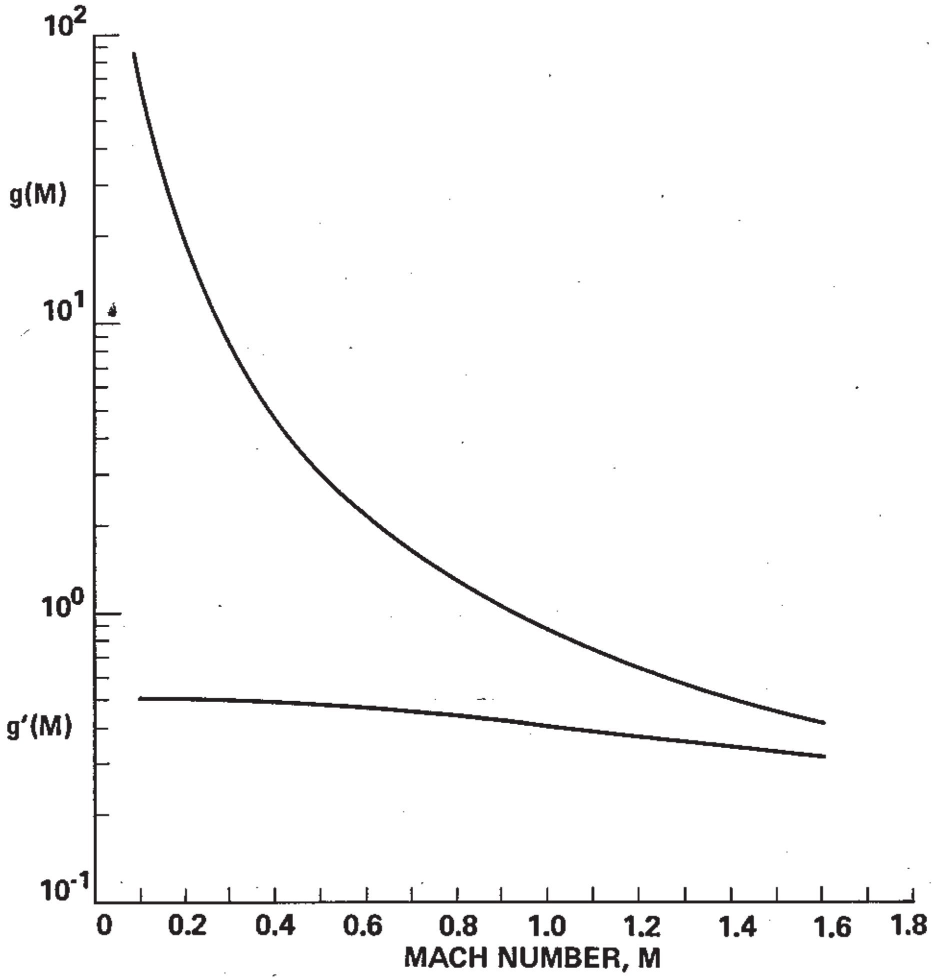


FIG. 4 COEFFICIENT LINKING ERROR IN CALCULATED VELOCITY WITH ERRORS IN MEASURED PRESSURES