ON THE DEFINITION OF THE AXIAL VELOCITY DENSITY RATIO IN THEORETICAL AND EXPERIMENTAL CASCADE INVESTIGATIONS

by

H.A. Schreiber and H. STARKEN
DFVLR Köln

Abstract

The Axial Velocity Density Ratio (AVDR) is one of the important parameters in the analysis of turbomachinary blade element and cascade flow. This parameter has a strong influence on the blade pressure distribution and the blade performance. The cascade data, which are obtained either from experiments in different cascade test facilities or from numerical calculations are comparable only, when the definitions of the AVDR are based on the same assumptions.

This contribution presents some examples, by which the influence of the AVDR on the blade pressure distribution with different stream-wise flow contractions is discussed.

List of Symbols

b	stream sheet thickness
ℓ	chord length
M	Mach number
M^+	critical Mach number
p	static pressure
p_o	total pressure
t	blade spacing
T	temperature
w	flow velocity
β	flow angle (with respect to cascade front)
β_s	stagger angle

1. Introduction

In general the flow through an axial flow turbomachine is calculated by dividing the core-flow into axisymmetric stream sheets. However, according to the different shape of hub and casing and different design philosophies the thicknesses of these stream sheets are changing in axial direction. This results in a variation of the axial velocity density distribution through the blade row. The axial velocity density distribution, however, has a considerable influence on the blade element performance. It changes the blade surface velocity distribution, the static pressure ratio, and the flow turning.

If the blade element flow is simulated with the help of the cascade model, the influence of the stream sheet thickness variation must be simulated too (Fig. 1). This can be done by varying the shape of the sidewall contours using wedge type inserts or flexible sidewalls in the cascade passage [1, 2], or by suction through the sidewalls of the windtunnel [3, 4, 5] .

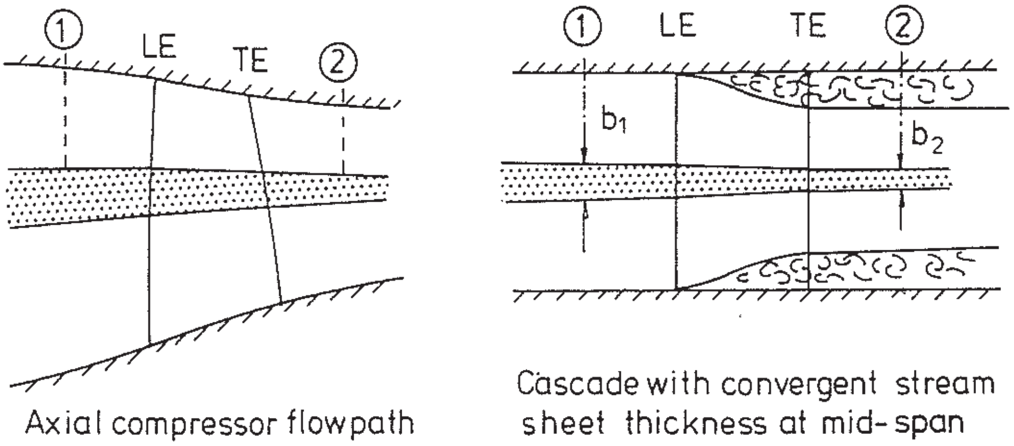

Fig. 1 Stream sheet variation within an axial flow compressor and the cascade model

Already with parallel sidewalls, the growth of the wall boundary layer results in an increase of the axial velocity density product ρw_{ax} at mid-span.

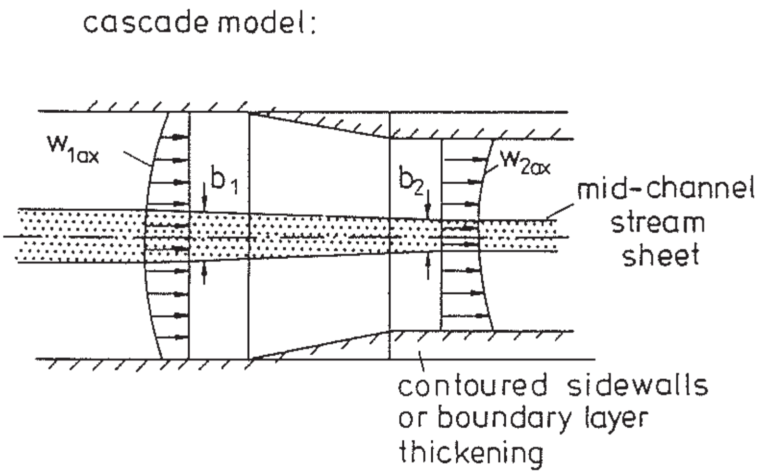

Fig. 2 Cascade model with convergent sidewalls and stream sheet distribution at mid-span

In reality the flow through a rectilinear cascade in a windtunnel with contoured sidewalls or with boundary layer thickening on the sidewalls is more or less three-dimensional, even if one would neglect secondary flow effects (Fig. 2). Likewise, the flow along the streamsurface of an axial flow turbomachine is three-dimensional because in reality the streamsurface is not axisymmetric.

However, to enable an adequate investigation of such a 3-dim. blade to blade flow with varying stream sheet thickness in cascades or in the turbomachine it is useful to introduce a simplifying parameter. This parameter shall describe the change of the stream sheet thickness or the change of the axial velocity density value throughout the blade row from an inlet to an outlet plane. This parameter is called the axial velocity density ratio (AVDR).

Defining this parameter, one has to be well aware of the fact, that there can be different types of axial velocity density distributions along the corresponding stream sheet. With different axial compressor designs there exist different axial velocity density distributions across the blade rows. Even with changing operating conditions of the turbomachine, the axial velocity density distribution can be different.

Concerning the investigation of cascades, the axial velocity density distribution along the mid-channel stream sheet of the windtunnel can be a funktion of the test conditions (cascade loading, incidence, pressure ratio) or of the test section geometry (for example aspect ratio of the blades [6,7]).

Knowing this, the designer of a turbomachine as well as the man who makes theoretical or experimental cascade investigations has to use the same definition of the axial velocity density ratio. The aim of this paper is to make some basic considerations to find a useful definition of the AVDR.

2. Definition

In rectilinear cascades the AVDR is defined for the mid-channel stream tube between a plane ① in front of the cascade and a plane ② behind the cascade (see Fig. 1 + 2). It is assumed, that the flow is quasi-two-dimensional and not influenced by secondary flow effects. The definition is derived from the continuity equation.

$$b_1 \rho_1 w_1 \sin\beta_1 = b_2 \rho_2 w_2 \sin\beta_2$$

$$\boxed{\Omega = \frac{b_1}{b_2} = \frac{\rho_2 w_2 \sin\beta_2}{\rho_1 w_1 \sin\beta_1}} \qquad \text{AVDR}$$

Because it is possible that there is a change of the axial velocity density even upstream or downstream of the blades, it is necessary to define the exact position of the reference planes ① and ② . Furthermore, the knowledge of the axial velocity density distribution is required.

3. Example: AVDR influence on the blade surface Mach number distribution

Figure 3 shows the sensitivity of the blade surface Mach number distribution of a compressor rotor blade cascade on the AVDR parameter. Presented are theoretical Mach number distributions calculated with the assumption of a linear stream sheet thickness distribution from the cascade leading edge plane to the trailing edge plane [8]. The AVDR values are varied between Ω = 1.0 (2-dim. flow conditions) and Ω = 1.1. As there is no change of the stream sheet thickness upstream or downstream of the cascade, the reference planes for the Ω - definition can be at any position upstream and downstream of the blades.

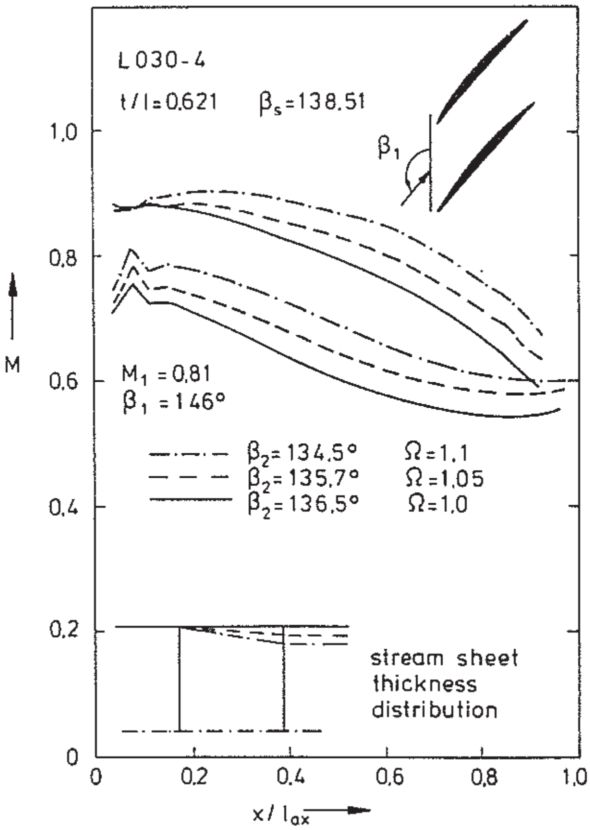

Fig. 3 Influence of the axial velocity density ratio Ω on the cascade blade surface Mach number distribution

The various downstream flow angles β_2, which are required for the calculation method, correspond to measuring values.

4. Influence of various stream sheet thickness distributions

The stream sheet thickness throughout the cascade can change not only within the blade passage but also in front or behind the blades. From measurements we know, that the stream sheet thickness at mid-span and thus the axial velocity density distribution can vary between the upstream and downstream measuring planes. Different kinds of this stream sheet thickness distribution, however, result in different blade surface pressure distributions, even for the same overall axial contraction ratio from a far upstream plane to a far downstream plane.

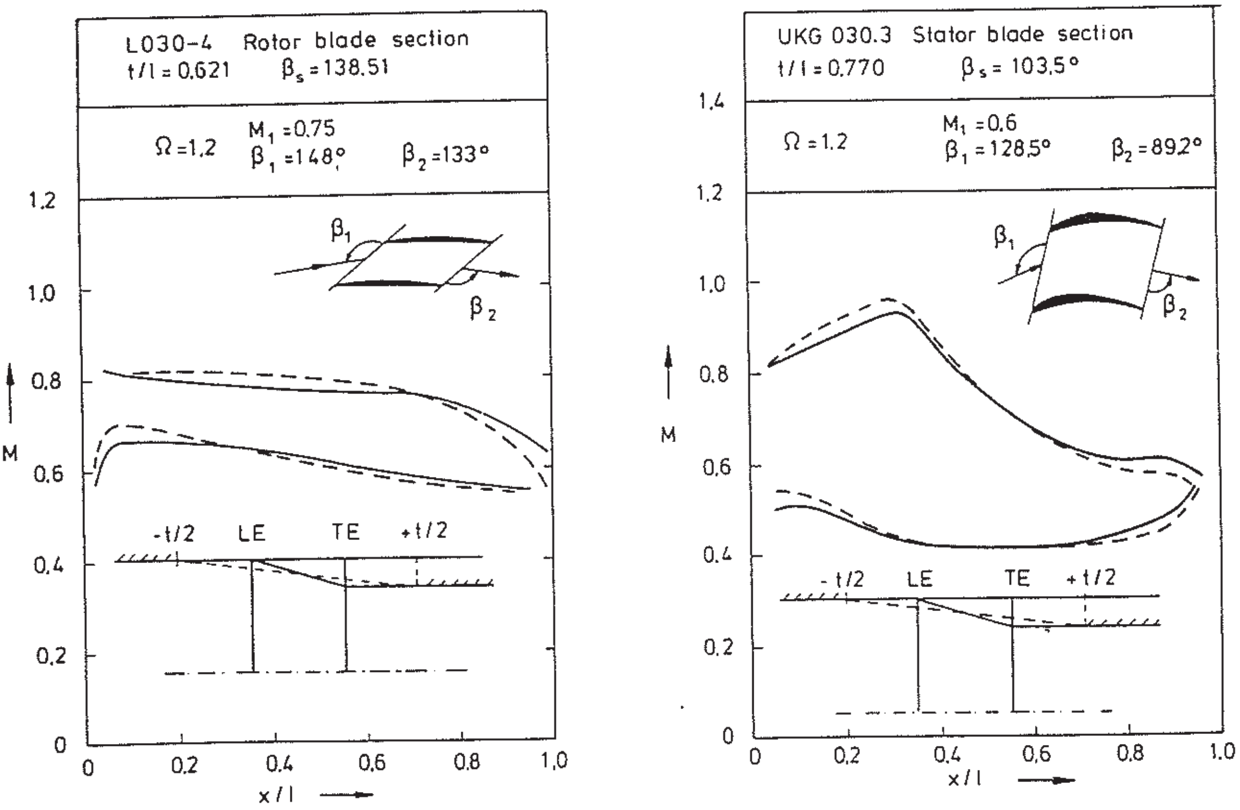

Fig. 4a

Fig. 4b

Influence of the stream sheet thickness distribution on the blade surface Mach numbers at constant overall axial contraction ratio of 1.2 (corresponds to AVDR = 1.2 between far upstream and far downstream)

Two examples in Figure 4a and 4b shall illustrate this: Calculated blade surface Mach number distributions of a rotor blade and a stator blade section are presented [9]. In both examples the overall axial contraction and thus the axial velocity density ratio, - in this case defined between planes far upstream and downstream -, is constant ($\Omega = 1.2$). The axial stream sheet distribution, however, is different. In the first case (dashed lines), a linear contraction is assumed between a plane half a gap upstream to half a gap downstream. These planes usually correspond to the measuring planes in cascade tests. In the second case the linear contraction is concentrated to the blade passage from the leading edge plane to the trailing edge plane.

The difference in the blade surface Mach number distribution is obvious. The contraction from half a gap upstream to half a gap downstream results in higher surface velocities in the front part of the blades and lower velocities in the rear part of the blades (dashed lines). Simultaneously the pressure rise gradients on the blade suction and pressure surface become steeper in comparison to the case with contraction only within the blade passage.

5. Change of the flow vectors upstream and downstream of the blade row due to variations in stream sheet thickness

A variation of the stream sheet thickness upstream and downstream of the blade row forces a change of the flow vectors. A contraction in axial direction for example, as shown in Figure 5, causes an increase of the velocities and a reduction of the flow angles. Figure 5 illustrates the change of the critical Mach numbers and flow angles upstream and downstream of the blades. A change of the stream sheet thickness and thus a change of the axial velocity density does not influence the circumferential component of the velocity or the circumferential component of the critical Mach number.

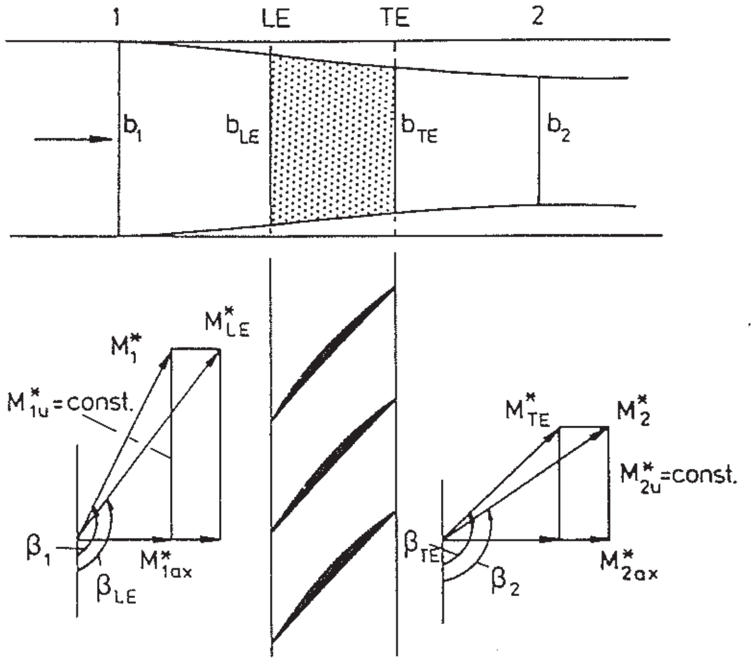

Fig. 5 Change of the flow vectors upstream and downstream of a cascade due to axial contraction

With the help of the continuity equation and the assumption of a constant circumferential velocity outside of the blade row a calculation of the velocity and flow angle change easily is possible.

The relations for flow values between an upstream plane ① and the cascade leading edge plane (LE) are as follows:

(1) $M^*_{1u} = \text{const} : \quad M^*_{LE} \cos\beta_{LE} = M^*_1 \cos\beta_1$

(2) (cont) $\left(\frac{\rho}{\rho_o}\right)_{LE} M^*_{LE} \sin\beta_{LE} = \frac{b_1}{b_{LE}} \left(\frac{\rho}{\rho_o}\right)_1 M^*_1 \sin\beta_1$

with b_1 and b_{LE} as stream sheet thicknesses in plane 1 and LE

Analogous for the downstream region we get:

$$(3) \quad M^*_{2u} = \text{const} \quad : \quad M^*_{TE} \cos\beta_{TE} = M^*_2 \cos\beta_2$$

$$(4) \quad \text{(cont)} \quad \left(\frac{\rho}{\rho_o}\right)_{TE} M^*_{TE} \sin\beta_{TE} = \frac{b_2}{b_{TE}} \left(\frac{\rho}{\rho_o}\right)_2 M^*_2 \sin\beta_2$$

The local stream sheet thicknesses b we get from the axial velocity density ratio determined in the measuring planes ① and ②

$$\frac{b_1}{b_2} = \Omega_{12}$$

and an assumed stream sheet thickness variation.
The change of Mach number and flow angle between plane ① and (LE) and plane (TE) and ② is illustrated in Figure 6.
Plane ① and ② are located one gap upstream of plane (LE) and one gap downstream of (TE) . For simplification again a linear contraction was assumed. The calculated flow data are summarized in Table 1.

Rotor blade cascade LO30-4 $t/\ell=0.621$ $\beta_s=138.51^\circ$

Station	1	1'	LE	TE	2'	2
M	0.807	0.815	0.823	0.595	0.607	0.612
β	146,2°	145.50°	144.79°	135.45	134.43°	133,98°

Ω_{12} = 1,14 $\Omega_{1'-2'}$ = 1.092 Ω_{LE-TE} = 1.0464

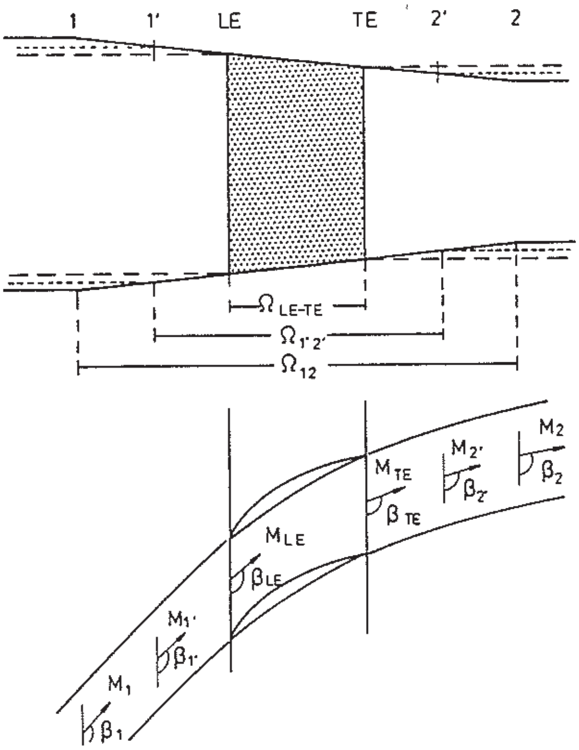

Fig. 6 Mach number and flow angle variation upstream and downstream of a cascade

6. Blade surface velocity distribution with stream sheet thickness variation upstream and downstream of the blade row

To demonstrate, whether the stream sheet thickness variation outside of the blade passage has any influence on the blade surface velocity or Mach number distribution respectively, some numerical experiments have been carried out.

As in the foregoing example a linear contraction was assumed. The stream sheet thickness distribution within the blade passage was kept constant as shown in Figures 7a and 7b where also Mach numbers, flow angles and contraction ratios (Ω) in the reference planes are indicated.

In the first case Ⓐ the contraction was applied between one gap upstream and one gap downstream. In the second case Ⓑ there was no change of the stream sheet thickness in front or behind the blades.

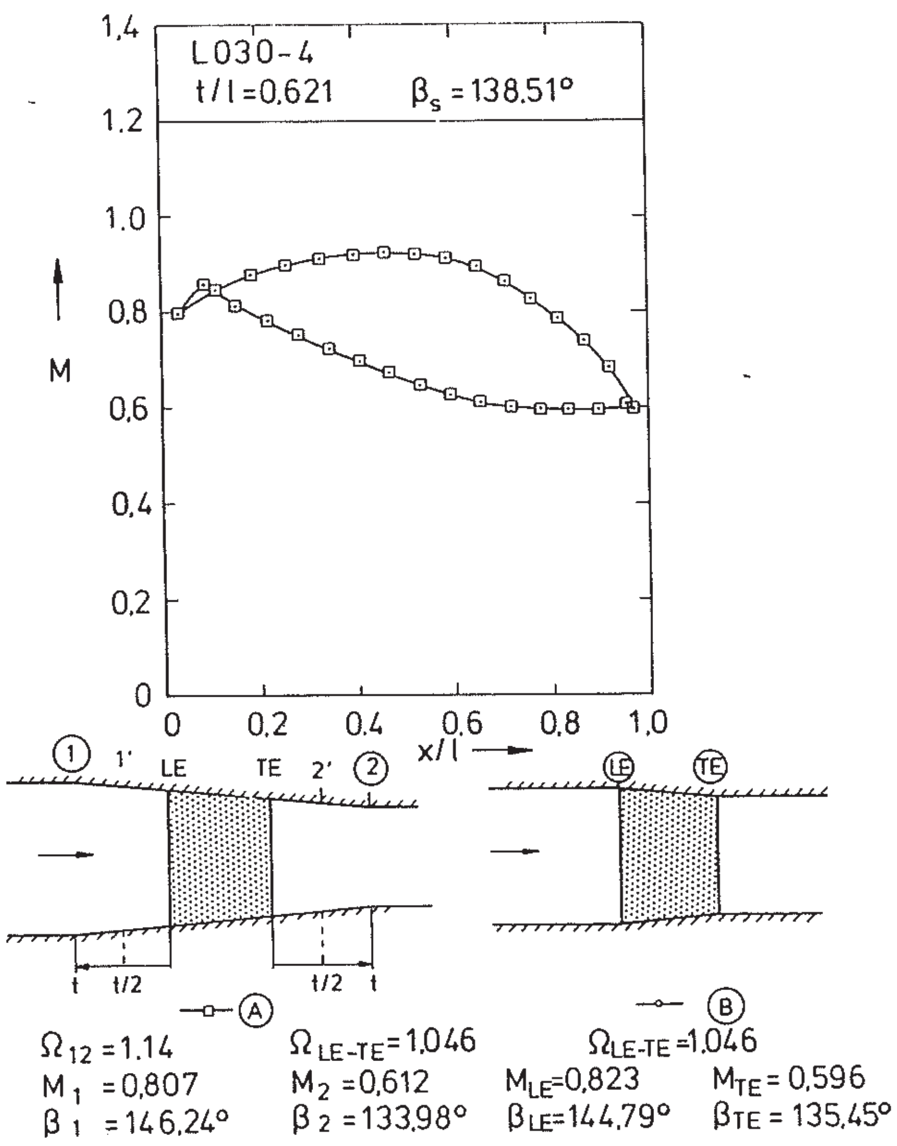

Fig. 7a

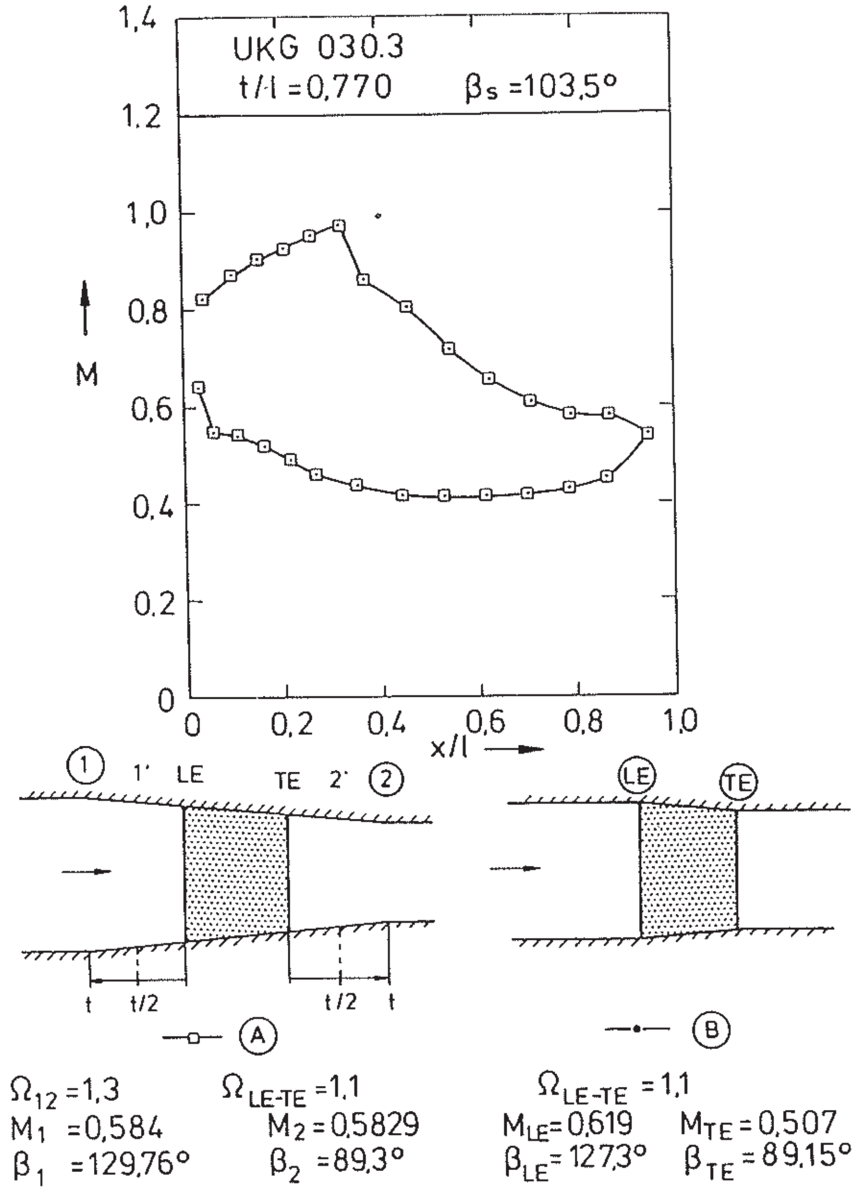

Fig. 7b

Comparison of the blade Mach number distributions with contraction between plane 1 and 2 (case Ⓐ) and contraction between LE and TE only (case B)

The result of the quasi-two-dimensional numerical calculations show, that the blade surface Mach number distributions of both examples are absolutely identical although the flow vectors far upstream and downstream of the cascades are different.

From this we can conclude that a stream sheet thickness variation only within the blade passage does influence the blade surface velocities and thus the boundary layer development and loss production. Thus a definition of the axial velocity density ratio seems to be useful only between the leading edge plane LE and the trailing edge plane TE .

$$\boxed{\Omega = \frac{\rho_{TE}\, w_{TE}\, \sin\beta_{TE}}{\rho_{LE}\, w_{LE}\, \sin\beta_{LE}}}$$

The values at LE and TE, however, have to be determined as averaged values.
Any change of the axial velocity density upstream or downstream changes the flow vectors, indeed, but does not vary the flow field within the blade passe. The corresponding flow vectors upstream and downstream can be calculated easily by the equations of chapter 5.

7. Axial velocity density distribution in cascade windtunnel tests

The stream sheet thickness distribution at cascade mid-span and thus the axial velocity density distribution through the blade row depends on various test parameters and the test section geometry. The aspect ratio of the blades for example can have an influence on the axial velocity density distribution at mid-span as is shown in [6,7]. But also the blade geometry, the blade loading (flow turning and pressure ratio) and the test conditions (whether wedge type inserts, parallel side walls or side wall suction are used) can change the side wall conditions and thus the axial velocity density distribution at mid-span.

Two experiments performed in the transonic cascade windtunnel of the DFVLR in Köln-Wahn have been used for comparison with calculations [8], to find out the actual mid-span stream sheet thickness distribution through the blade row (Figure 8 and 9). Both tests have been run with a side wall suction device. The blade aspect ratio was 1.86 and 2,38 respectively. The Mach numbers and flow angles are determined in the measuring planes approximately half a gap upstream and half a gap downstream.

In both examples a relatively good agreement of theory and experiment was obtained with an assumption of a linear contraction from leading edge to trailing edge plane. Other experiments, however, may show different stream sheet thickness distribution between the measuring planes.

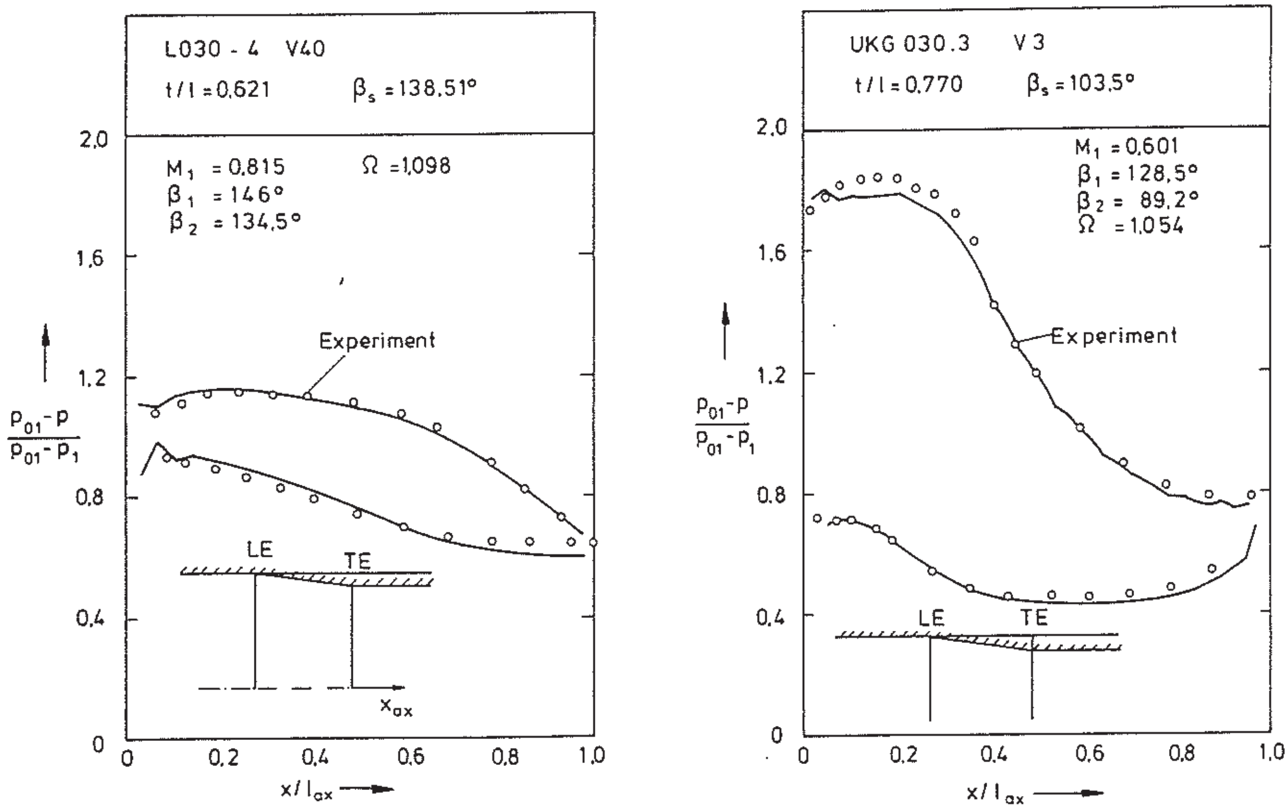

Fig. 8 and Fig. 9
Comparison of experimental and theoretical blade pressure distribution assuming a linear axial velocity density increase between LE and TE.

But it must be emphasized that it is very difficult, or nearly impossible to find the exact experimental stream sheet thickness distribution from the comparison of measured and calculated surface pressure distributions. The change of the blade pressure distribution due to variations in the stream sheet thickness distributions is already in the same order of magnitude as the variations due to errors in measurements.

8. Conclusions

The paper attempted to find a useful definition of the AVDR. The main problem was to determine relevant reference planes for the up-stream and downstream flow values.

With the help of some numerical calculations it has been shown, that the blade surface velocity distribution is not influenced by any change of the axial velocity density value upstream or downstream of the blade row. Thus it seems logical to define the AVDR only between the leading edge plane and the trailing edge plane. With the help of this definition, data obtained from cascade experiments or calculations can be transferred easily to corresponding stream surfaces of axial flow turbomachines.

9. References

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