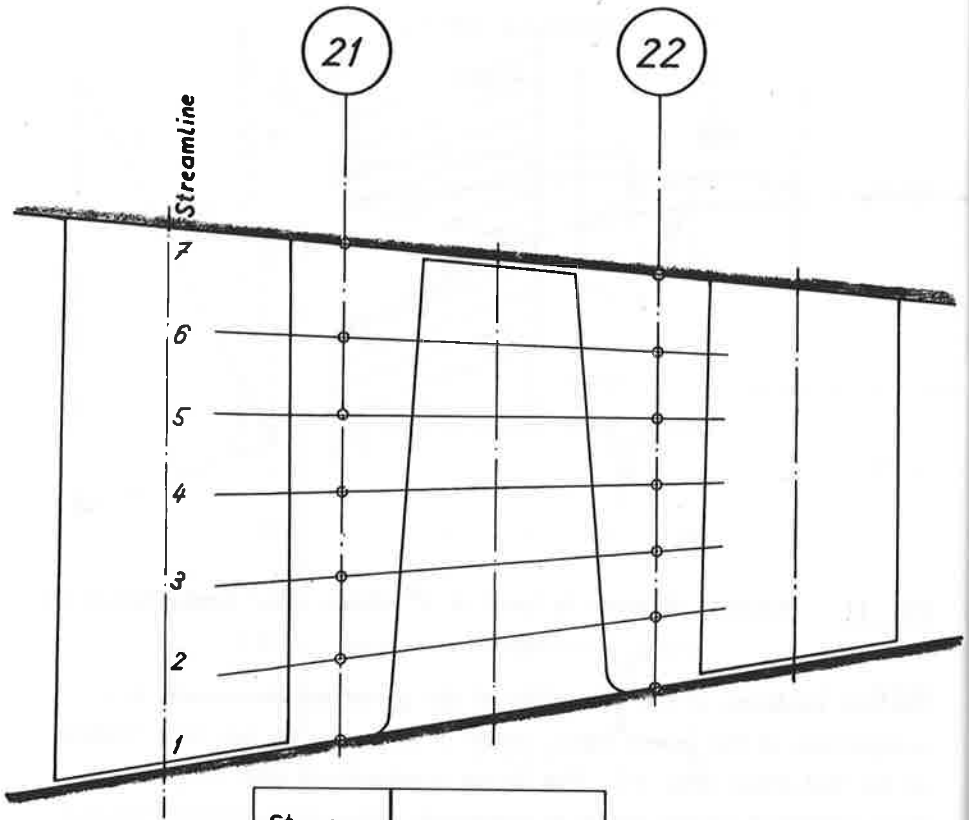

Streamline	$\dfrac{\Delta T_t - \Delta T_{EULER}}{\Delta T_t}$
2	- 0.0234
3	- 0.0129
4	+ 0.0172
5	+ 0.0489
6	- 0.0029

ΔT_t = measured value

$$\Delta T_{EULER} = \frac{u_{22} c_{u22} - u_{21} c_{u21}}{c_p}$$

Fig. 15 Measured total temperature rise compared with temperature rise calculated from velocity diagrams (2^{nd} stage rotor)

Multi-parameter approximation of calibrating values for multi-hole probes

by

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In this contribution there is presented a method for calculating the calibration spheres and calibration spaces of subsonic and supersonic probes by multiparameter approximation functions.

To determine the flow vector in both magnitude and direction, probes are normally used, which have special features, due to the measuring task.

In Fig. 1 there are shown the two types of five-hole-probes used for our measurements: On the left hand a semi-spherical five-hole-probe, normally adapted for subsonic measurements, and on the right hand, a conical five-hole-probe, used for supersonic flow measurements. Beneath the probe head, there are located two NTC-thermistors for measuring the flow temperature.

The probes presented here deliver seven informations independent from each other. Thus, seven flow parameters can be uniquely determined, as follows:

total pressure p_t
static pressure p
flow velocity c (Mach-number M)
the two angles of flow vector α, β
total temperature T_t
static temperature T

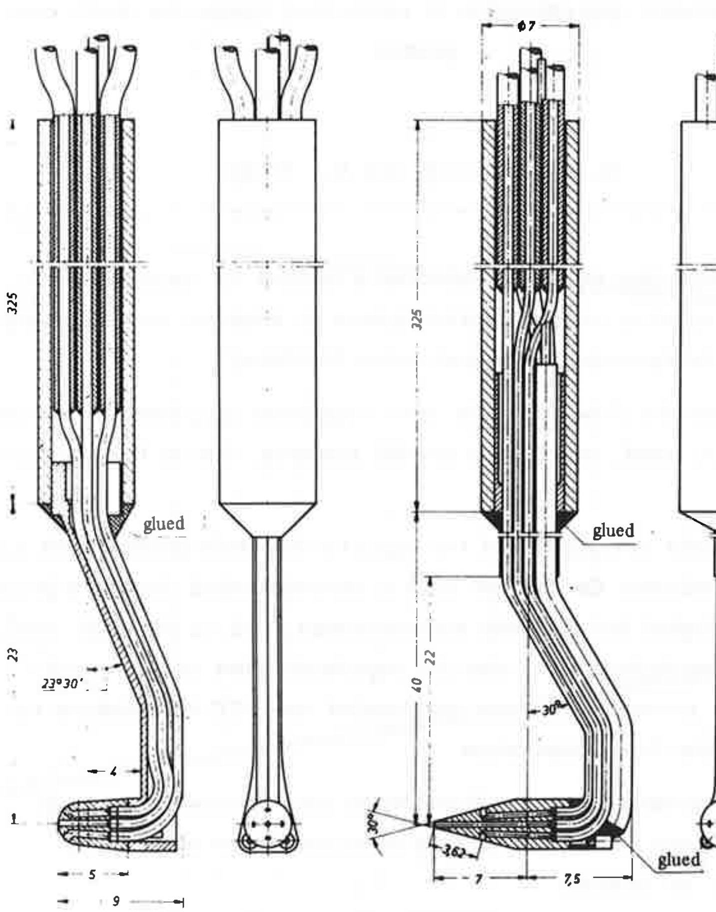

Fig. 1 Subsonic and supersonic five-hole-probe

Fig. 2 shows the location and the numeration of the pressure holes and the direction of the flow vector at the probe head.

Due to the calibration data, functions are to be stated now to compute the above mentioned flow parameters. As the measured calibration values are only depending on the variables Machnumber and the angles

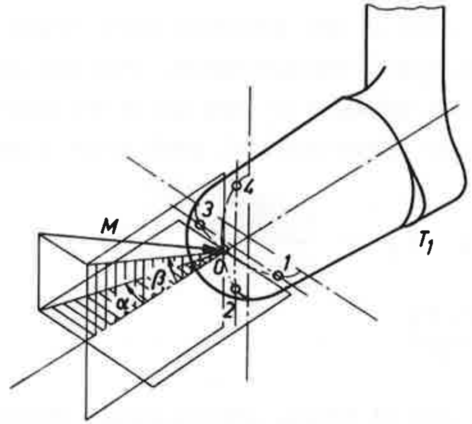

Fig. 2 Probe head

α and β, there must exist a unique interdependency between these variables and the calibration data, on the other hand.

In Fig. 3 the calibration space of a five-hole-probe is plotted.

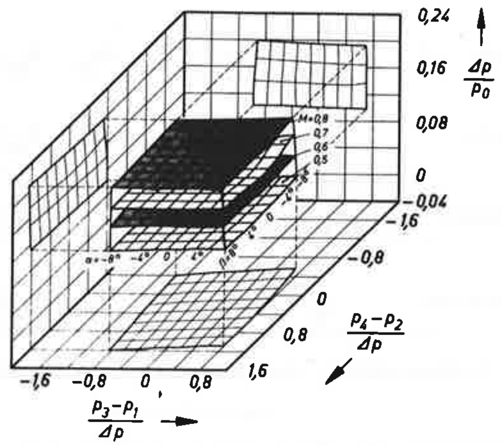

Fig. 3 Calibration space of a five-hole subsonic probe

To enable the description of this interdependency, there have to be established non-dimensional characteristics. They are required to represent the essential influence of each one of the following flow variables: flow velocity (Mach-number), pitch angle α and yaw angle β:

$$k_\alpha = \frac{p_3-p_1}{\Delta p}; \quad k_\beta = \frac{p_4-p_2}{\Delta p}; \quad k_M = \frac{\Delta p}{p_o}$$

$$\Delta p = p_o - \frac{p_1+p_3}{2}$$

The characteristics defined before, advantageously depend only on the pressure values measured by the probe head.

There can be established now the following functions for the flow parameters by the aid of the defined characteristics:

pitch angle	$\alpha = f_1 (k_\alpha, k_\beta, k_M)$
yaw angle	$\beta = f_2 (k_\alpha, k_\beta, k_M)$
Mach number	$M = f_3 (k_\alpha, k_\beta, k_M)$
total pressure	$\frac{p_t-p_o}{\Delta p} = f_4 (k_\alpha, k_\beta, k_M)$
static pressure	$\frac{p_o-p}{\Delta p} = f_5 (k_\alpha, k_\beta, k_M)$
total temperature	$\frac{T_{g1}-T}{T_t-T} = f_6 (k_\alpha, k_\beta, k_M)$
static temperature.	$\frac{T_{g2}-T}{T_t-T} = f_7 (k_\alpha, k_\beta, k_M)$

T_{g1}, T_{g2} - Temperatures, measured with the thermistors

The further mathematical procedure may be explained by the aid of the map of the angle α (Fig. 4): Within the spatial coordinate system, established by the independent characteristics k_α, k_β, (k_M = const), a sphere is built up by the calibration data. The calibration function of the pitch angle $\alpha = f (k_\alpha, k_\beta, k_M)$

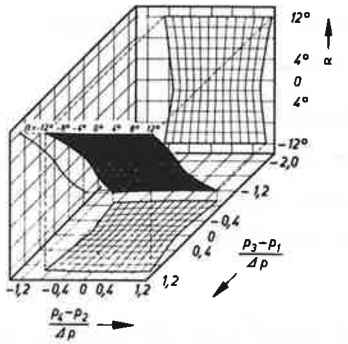

Fig. 4 Calibration sphere of a five-hole subsonic probe (k_M = const.)

replaces this sphere, since the grid points of corresponding discrete sections in the three directions are to be interpreted as angle data. For the mathematical formulation of the mentioned function a polynom-expression is chosen which includes the three independent characteristics:

$$\alpha = \sum_{i=1}^{l} \sum_{j=1}^{m} \sum_{k=1}^{n} a_{ijk} \, k_M^{\,k-1} \, k_\beta^{\,j-1} \, k_\alpha^{\,i-1}$$

a_{ijk} ..... unknown coefficients
l m n ..... total number of coefficients

Different mathematical procedures are well-known to determine this function from measured calibration data.

In principle, there exist two different procedures:

1) the method of interpolation
2) " " " approximation

The interpolation methods use functions which fulfill the ordinate value in each measured point exactly. On the contrary, by aid of the approximation methods functions are determined which fairly substitute the system of the measured values without replacing the measured values.

Two approximation conditions are of importance for practice:

1) the Tschebyscheff Minimum Condition
2) the Gaussian Minimum Condition

According to Tschebyscheff, there are determined functions of that kind that the maximum error becomes a minimum. The application of these functions very often meets heavy formal difficulties, as the system of equations to determine the unknown coefficients, is non-linear.

These difficulties don't arise with the Gaussian condition, which requires the error of the approximative functions has to become a minimum. Due to the Gaussian condition a set of linear equations for the unknown coefficients can be built up. The Gaussian Minimum Condition has been applied to the calculation of the calibration values of five-hole-probes. The Minimum Conditions is as follows:

$$Q = \sum_{r=1}^{u} \sum_{s=1}^{v} \sum_{t=1}^{w} \left[\left(\sum_{i=1}^{l} \sum_{j=1}^{m} \sum_{k=1}^{n} a_{ijk} \, k_M^{\,k-1} \, k_\beta^{\,j-1} \, k_\alpha^{\,i-1} \right) - \alpha_{rst} \right]^2 = \mathrm{Min}$$

$u \cdot v \cdot w$ ..... total number of calibration data

α_{rst} ..... calibration data

Q ..... least mean square

The partial differentiation of this equation after all unknown coefficients

$$\frac{\partial Q}{\partial a_{111}} = 0 \; ; \quad \frac{\partial Q}{\partial a_{112}} = 0 \; \ldots \; \frac{\partial Q}{\partial a_{1mn}} = 0$$

results in the necessary number of independent equations for the computation of the coefficients. This system of equation can be solved by the Elimination-Method or other well-known numerical methods.

In Fig. 5 there are demonstrated some results of the applications of the multi-parameter approximation for different five-hole-probes.

Sondentyp	Machzahl	Anzahl der Eichwerte	Funktion	Anzahl der Koeffizienten	$\bar{F}_P$	$\bar{F}_A$
5-Loch-Sonde (feststehend)	0,4 ÷ 0,8	324	α [°]	48	0,048	0,0026
			β [°]	48	0,053	0,0029
			M	48	0,0074	0,0004
			$(p_t - p_0)/\Delta p$	48	0,0017	0,0001
			$(p_0 - p)/\Delta p$	48	0,015	0,0008
			$(T_{g1} - T)/(T_t - T)$	64	0,0079	0,0004
			$(T_{g2} - T)/(T_t - T)$	64	0,0048	0,0002
5-Loch-Sonde (nachdrehbar)	1,2 ÷ 1,8	54	β [°]	9	0,26	0,036
			M	9	0,021	0.0028
			$(p_t - p_0)/\Delta p$	9	0,036	0,0049
			$(p_0 - p)/\Delta p$	9	0,0063	0,0008
			$(T_{g1} - T)/(T_t - T)$	16	0,0033	0,0004
			$(T_{g1} - T)/(T_t - T)$	16	0,0023	0,0003
5-Loch-Sonde (feststehend, inkompressible Strömung)	0,4	169	α [°]	25	0,0680	0,0052
			β [°]	25	0,1020	0,0079
			$(p_t - p_0)/\Delta p$	25	0,0025	0,0002
			$(p_0 - p)/\Delta p$	25	0,0015	0,0029

Fig. 5 Survey of the errors by application of the approximative functions

$$\bar{F}_P = \sqrt{\frac{Q}{u \cdot v \cdot w - 1}} \; ; \qquad \bar{F}_A = \frac{\bar{F}_P}{\sqrt{u \cdot v \cdot w}}$$

CONCLUSION

By the aid of multi-parameter approximation there can be replaced calibration spheres and calibration spaces by only one function. The coefficients of such a function being known, the calculation of data measured by five-hole-probes in the flow field is considered quick and easy. No interpolation and iteration is needed, to determine the flow characteristics.